The 20th International Conference on
Transport and Sedimentation of Solid Particles
26–29th September 2023, Wroclaw, Poland

ISSN 0867-7964 ISBN 978-83-7717-392-3

TOWARDS THE ASSESSMENT OF THE PREDICTIVE CAPACITY OF THE *β-σ* TWO-FLUID MODEL FOR PSEUDO-HOMOGENEOUS SLURRY FLOW IN PIPES

Qi Yang[1,2], Gianandrea V. Messa[1*] and Václav Matoušek[3]

DOI: 10.30825/4.14-11.2023

*[1]Dept. Civil and Environmental Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy. * e-mail: gianandreavittorio.messa@polimi.it*
[2]Dept. Hydraulic Engineering, Tsinghua University, Shuangqing 30, 100084 Beijing, China. e-mail: qi.yang@polimi.it
[3]Dept. Civil Engineering, Czech Technical University in Prague, Thákurova 7, 166 29 Prague 6, Czech Republic. e-mail: v.matousek@fsv.cvut.cz

ABSTRACT: This paper focuses on the numerical simulation of turbulent, pseudo-homogeneous slurry flows in pipes through the *β-σ* two-fluid model, developed by the authors and collaborators in previous research. The two-fluid model gives its name to the presence of two main calibration coefficients, namely, *σ*, associated with the turbulent dispersion of the particles, and *β*, related to the inter-phase friction and to the wall shear stress produced by the solid phase. In a recently published article, the role played by *β* and *σ* on different features of the CFD solution has been established for different flow conditions, and a procedure for the calibration of the two coefficients has been proposed. The present contribution investigates the extrapolability of previously calibrated coefficients to different conditions in terms of pipe diameter, particle type, and in-situ concentration. The experimental data used to support the conclusions and recommendations from the numerical study were obtained from previously published literature. The findings of this study not only contribute to a deeper comprehension of the *β-σ* two-fluid model, but they also provide a methodological background for the development of computational tools for industrial practitioners and academic researchers.

KEY WORDS: slurry pipe flow, Computational Fluid Dynamics, two-fluid modeling, calibration and validation

1. INTRODUCTION

Hydrotransport of solid particles in the form of a slurry is widely adopted in various industrial applications such as mineral processing, oil and gas transportation, and wastewater treatment. Slurries, which consist of a mixture of solid particles in a carrier liquid, exhibit complex dynamics resulting from the interactions among the particles, the liquid, and the pipe walls. Understanding and, more importantly, being able to predict these intricate physical mechanisms is of paramount importance for ensuring the efficient and reliable operation of these processes. Consequently, this challenge has received significant attention over the past few decades from both the academic community and the industrial R&D sector.

Several approaches have been proposed and are still widely used for the prediction of slurry flow in pipelines, including empirical or semi-empirical models obtained from field observation and laboratory testing, mechanistically-based models (e.g., layered models), and numerical modelling based on Computational Fluid Dynamics (CFD) simulations. Thanks to the rapid development of commercial codes for engineering simulation, CFD has become an increasingly important approach in hydro-transport applications; in a recent review paper on this topic (Messa *et al.* 2021), we found that about 60 research articles dedicated to this topic have been published in scientific journals since 2000. The added value of CFD compared to alternative investigation methods relies on the fact that this approach relies on the modelling of the fundamental fluid dynamic processes at the local level and, therefore, this method is capable of providing extensive distributed information. Some of the parameters that can be calculated through a CFD simulation are challenging to measure but relevant from an engineering perspective; these include, for instance, the distribution of the particle impact velocities against the pipe wall, which significantly affect the wear phenomenon. Very important, the local description of flows makes – in principle – CFD free from any constraint in terms of the size and geometric complexity of the system under investigation. The considerations here above indicate that, in hydro-transport applications, CFD can have a real practical impact for (i) estimating difficult-to-measure parameters in easy-to-test systems, e.g., the detailed wall shear stresses distributions in a small slurry pipeline and (ii) gathering information on difficult-to-test systems, e.g., a large-diameter pipeline or complex pipeline components. However, making CFD an effective engineering tool is not a straightforward task, in spite of the capabilities and user-friendly features of commercial codes. The computational cost of CFD simulations is high, and in order to keep it within acceptable limits, and fundamental equations derived from the basic physical principles are manipulated and approximated before being solved numerically. This produces a number of sources of uncertainty, taking the form of difficult-to-decide sub-models, closures, and coefficients, for which, practically speaking, no other selection criterion is available rather than the calibration based on experimental data. Therefore, in order to gain confidence in the predictions of a CFD model, one must: (i) identify such "modelling" sources of uncertainty, assessing their role on the different features of the CFD solution and thus determining their relative importance; (ii) perform a calibration of the model based on experimental data, which implies determining which number and type of data are needed and disposing of a rigorous calibration strategy; (iii) establish the accuracy of the solution outside the calibration condition, setting the limits of validity of the calibrated model. Note that the uncertainty produced by these "modelling" factors (which require calibration) combines with the error arising from the numerical solution of the manipulated flow equations, which must be controlled through appropriate convergence studies. And, clearly, also the uncertainty of the measurements used for the calibration and the validation of the model must be accounted for, as significant inaccuracies might affect the quality of solids concentration and mixture velocity data.

In our last co-authored publication (Messa *et al.* 2023), we discussed items (i) and (ii) for the β-σ two-fluid model, which is a CFD model based on the Eulerian-Eulerian approach that we developed together with other collaborators for the simulation of turbulent slurry pipe flows of fine particles, in which the particle transport is dominated by their interaction with the turbulent flow (the so called "pseudo-homogeneous" regime according to Wilson *et al.* 2006). The focus of the present study is item (iii) We investigated whether the values of the two main calibration coefficients of the β-σ two-fluid model, namely, β and σ, were appropriate also outside the calibration range, specifically for larger pipe diameters and different particle materials. If this was not the case, we sought for possible trends for the extrapolation of the two coefficients. This work makes a contribution towards accomplishing task (iii), which is of paramount importance to make the β-σ two-fluid model a useful tool for situations where experimental data are not attainable, primarily in difficult-to-test systems. Nonetheless, the scope of this study was mainly methodological in nature, as the same approach is recommended regardless of the specific CFD model used.

2. THE $\boldsymbol{\beta}$-$\boldsymbol{\sigma}$ TWO-FLUID MODEL

The complete formulation of the β-σ two-fluid model is reported in Messa and Matoušek (2020), and this should be the main reference for readers interested in all mathematical details. In this short paper, we will limit ourselves to review the essential aspects of the model.

2.1. ESSENTIAL FEATURES OF THE $\boldsymbol{\beta}$-$\boldsymbol{\sigma}$ FORMULATION

The β-σ model arises as an extension of the Inter-Phase Slip Algorithm (IPSA) of Spalding (1981) which, through appropriate modifications to the closure equations and the boundary conditions, expands its applicability to slurry flows in the pseudo-homogeneous regime. It applies to turbulent flows only. The formulation of the fundamental mass and momentum conservation equations is basically inherited from the IPSA and a distinctive feature is the presence of phase diffusion terms in all conservation equations, as follows:

$$\nabla \cdot \left(\rho_k \frac{\mu_\mathrm{l}^\mathrm{t}}{\rho_\mathrm{l}\sigma} \psi_k \nabla \Phi_k \right) \tag{1}$$

where $k = \mathrm{l}, \mathrm{s}$ represents the liquid and the solid phase, respectively; ρ is the density; μ^t is the eddy viscosity; Φ is the volume fraction; σ is the turbulent Schmidt number for volume fractions; and ψ is the generic transported variable, namely 1 in the mass conservation equations, the velocity vectors $\boldsymbol{U}$ in the momentum conservation equations, and the turbulent kinetic energy of the fluid, k, and its dissipation rate, ε, in the turbulence model equations. The formulation of the IPSA on which the β-σ two-fluid model relies can be interpreted as a two-phase analogous of the Reynolds-Averaged Navier-Stokes (RANS) in statistically-steady, turbulent single-phase flow. In fact, the variables $\boldsymbol{U}$, Φ and the pressure P can be regarded as the time-average of the locally volume averaged values; additionally, the eddy viscosity μ^t is introduced to model the correlations between the fluctuating velocity components. In this perspective, the phase diffusion terms (Eq. 1) arise from the modelling of the correlation between the fluctuating velocity and the fluctuating volume fraction. These terms account for the turbulent dispersion of the solid particles in the turbulent flow, which is governed by the turbulent Schmidt number for volume fraction; the lower the value of σ, the stronger the effect of the turbulent fluid fluctuations on the particle motion.

A peculiar feature of the *β*-*σ* two-fluid model, not inherited from the IPSA, is the introduction of a friction parameter, μ_m, function of the local solid volume fraction Φ_s through an empirical coefficient β, as follows,

$$\mu_\mathrm{m} = \mu_\mathrm{l}\exp\left\{\frac{2.5}{\beta}\left[\frac{1}{(1-\Phi_\mathrm{s})^{\beta}} - 1\right]\right\} \tag{2}$$

where μ_l is the viscosity of the liquid phase. The parameter μ_m appears in the equations of the *β*-*σ* two-fluid model three times. Firstly, μ_m is used in the evaluation of the viscosity of the solid phase, μ_s. Particularly, it is assumed that μ_m can be calculated as some sort of weighted average of μ_l and μ_s, the weights being the corresponding volume fractions Φ_s and Φ_l. In mathematical terms, this means that

$$\mu_\mathrm{m} = \Phi_\mathrm{l}\mu_\mathrm{l} + \Phi_\mathrm{s}\mu_\mathrm{s} \tag{3}$$

Secondly, μ_m plays a role in the calculation of the momentum transfer between the phases (inter-phase friction). In the *β*-*σ* two-fluid model, only the drag force is accounted for, and the drag coefficient, C_d, is expressed as a function of a friction Reynolds number Re_m, defined as

$$\mathrm{Re}_\mathrm{m} = \frac{\rho_\mathrm{l} d_\mathrm{p} |\boldsymbol{U}_\mathrm{l} - \boldsymbol{U}_\mathrm{s}|}{\mu_\mathrm{m}} \tag{4}$$

where d_p is the particle diameter. The functional relation between $\mathrm{C_d}$ and Re_m is formally analogous to the one developed by Schiller and Naumann (1935) to relate the drag coefficient and the particle Reynolds number for a single spherical particle in a uniform flow. Finally, μ_m appears in the evaluation of the wall shear stress exerted by the solid phase, which is obtained through some sort of analogue of the typical log-law formulation in single-phase flow simulations. As a final note, Eq. (2) is formally analogous to the comprehensive formula of Cheng and Law (2003) for the viscosity of the mixture of liquid-solid suspensions and, also for this reason, the same symbol μ_m was used to denote the friction parameter. However, the analogy is only formal, since, in the *β*-*σ* two-fluid model, the friction parameter cannot be interpreted as the viscosity of the mixture, as the particles are not extremely fine and their density is significantly higher than that of the carrier liquid.

2.2. APPLICABILITY CONDITIONS

The *β*-*σ* two-fluid model can only be applied under the following three conditions, which broadly speaking relate to the "pseudo-homogeneous" regime. An important feature of these constraints is that they can be verified a priori, that is, before running any simulation. The first condition pertains to the formulation of wall shear stress for the solid phase. It requires that the dimensionless particle diameter, d_p^{+}, to be less than 30, to that the log-law formulation mentioned at the end of the previous sub-section can be applied.

d_p^+ might be interpreted as some sort of particle Reynolds number defined with respect to d_p, ρ_l, μ_l, and the friction velocity of the liquid phase, U_l^*. Since the latter is an output of the CFD simulation, the well-known correlation of Blasius for single-phase flow was recommended for its evaluation. Thus, the condition $d_\mathrm{p}^+ < 30$ was replaced by $d_\mathrm{p}^{+\mathrm{B}} < 30$, where $d_\mathrm{p}^{+\mathrm{B}}$ is an estimate of d_p^+ in which U_l^* is obtained from the Blasius correlation for single-phase flows. In mathematical terms, this reads as follows,

$$d_\mathrm{p}^{+\mathrm{B}} = \frac{d_\mathrm{p}}{\frac{\mu_\mathrm{l}}{\rho_\mathrm{l} V_\mathrm{m}}\left[0.039\left(\frac{\rho_\mathrm{l} V_\mathrm{m} D}{\mu_\mathrm{l}}\right)^{-0.25}\right]^{-0.5}} < 30 \tag{5}$$

where D is the pipe diameter and V_m is the area-average velocity of the slurry, that is, the ratio between the total volumetric flow rate and the area of the pipe cross-section. Basically, Eq. (5) sets an upper limit to the range of V_m that, for a given combination of D and d_p, can be simulated through the *β-σ* two-fluid model.

The two other applicability conditions are related to the need for a fully-suspended flow, in which particle transport is dominated by the interaction between the solids and the turbulent liquid. The first constraint imposes that V_m must be higher than the deposition limit velocity, V_dl, which translates into the following practical condition:

$$V_\mathrm{m} > 1.5 V_\mathrm{dl}^\mathrm{T} \tag{6}$$

where V_dl^T is the estimate of the V_dl obtained from the formula of Thomas (2015). This condition sets a lower limit to the flow velocity and, basically, it reflects the incapability of the model to account for the particle-particle interactions occurring when particles start accumulating at the bottom of the pipe. The V_dl model of Thomas (2015) enlarged the applicability range of previous formulations to fine particles (down to 30 μm size) flowing in big pipes (up to 1000 mm diameter). The second constraint imposes an upper limit to the in-situ solid concentration, C_vi, equal to 0.40 to prevent a significant effect of particle-particle interactions even at high velocity, as obtained on the grounds of the experimental study of Korving (2002).

2.3. KEY CALIBRATION COEFFICIENTS AND CALIBRATION STRATEGY

As already mentioned, our last co-authored publication (Messa *et al.,* 2023) was focused on the handling of calibration coefficients of the *β-σ* two-fluid model. Although several other factors come into play, which were not explicitly investigated as their values could be regarded as well-established constants in fluid dynamic modelling, the main calibration parameters of the *β-σ* model are *β* and *σ*, which indeed give the name to the model itself. Particularly, the turbulent Schmidt number for volume fractions, *σ*, is associated with the modelling of the turbulent dispersion of the solid particles (the smaller the value of *σ*, the higher the impact of turbulent dispersion), whereas the other coefficient *β* relates to the modelling of concentration effects on interphase and wall friction.

The value of *σ* mainly affects the predicted concentration profile. Conversely, it has a very limited impact on the hydraulic gradient i_m, that is, the drop of hydraulic head per unit length of pipe, and practically no effect on the velocity distribution. Conversely, the value of β plays a role only at high solid concentration, and under this condition, it significantly affects the hydraulic gradient, with a relatively minor impact on the concentration profile and practically no effect on the velocity field. Based on these findings, a two-point procedure for deciding the values of β and σ through experimental calibration was proposed in Messa *et al.* (2023). This consists, firstly, in evaluating σ referring to a single concentration profile at moderate concentration, say $C_{\mathrm{vi}} \approx 10\%$, and, afterwards, deciding the appropriate β based on a single hydraulic gradient point at high concentration, say around 30-40%. The pair of values obtained from the calibration procedure were supposed and verified to be accurate within the range of C_{vi} of the two calibration points, and for V_{m} within the range defined by the first two applicability conditions of the β-σ model (Eqs. 5 and 6).

The two-point calibration procedure was successfully verified for three testing conditions, characterized by different pipe diameters and particle materials, and similar ranges of concentration (from about 10-15% to about 35-40%) and velocities (from about 1 to about 5 m/s). For each condition, the two-point calibration yielded different pairs of β and σ, which provided reasonable agreement with all the other data in the same dataset. Although the validation revealed some inherent limitations in the β-σ formulation, which appears uncapable in reproducing all the physical mechanisms driving the transport of massive amounts of particles, the good results and the simplicity of the model confirm its potential of being an engineering effective tool for slurry pipeline design.

3. SCOPE AND METHODOLOGY

The goal of this investigation is to investigate whether the combination of β and σ values obtained from the calibration is appropriate even outside the calibration range, focusing, in particular, on different pipe diameters and particle materials. Accomplishing this task required disposing of suitable sets of experimental data, which, in principle, should differentiate only for the parameter under investigation, namely, either the pipe diameter or the particle material. An extensive literature review allowed identifying a number of published experimental results that would be suitable for the purpose. Indeed, comparing data collected in different setups could be more challenging than referring to data coming from the same lab. In fact, several specific factors, which are difficult if not impossible to take into account in this investigation, might influence the comparison; these include, for instance, the configuration of the setups, the instrumentation used, the experimental protocol, the environmental conditions etc. Thus, the present study allowed identifying the overall behavior and trends of the β-σ two-fluid model solution and, possibly, planning new experimental tests to get deeper insight.

In some cases, no additional simulations were run, and the conclusions were drawn by analyzing, from a different perspective, the data already reported in Messa *et al.* (2023). In other cases, new simulations were required. If so, the CFD set up, intended as computational domain, boundary conditions, mesh and solution strategy (including under-relaxation settings and convergence criteria), was the same as reported in Messa *et al.* (2023). All simulations were run with the commercial CFD code PHOENICS 2018.

4. RESULTS AND DISCUSSION

The results regarding the effect of changing the pipe diameter and the particle material are presented hereafter in separate sections.

4.1. EXTRAPOLATION TO LARGER PIPE DIAMETERS

Although the slurry flow in large diameter pipes is of interest in many applications, like dredging, in the literature very few experimental data could be found regarding these processes. This is not surprising due to the challenges involved in conducting the tests, in terms of the amount of energy, water, and solids required. The investigation of large diameter pipes is therefore one of the cases in which CFD could have a practical impact by enabling virtual experiments. However, the scarcity of experimental data could make it difficult to perform the calibration of the model. Thus, we decided to investigate whether the calibrated values of β and σ obtained from laboratory tests on a small diameter pipe preserve their validity when a slurry with the same characteristics flows in a bigger pipe.

To this aim, we made reference to the experimental data reported in the PhD thesis of Gillies (1993), who conducted sand slurry tests in three horizontal pipes of different diameters, namely, 53.2 mm, 159 mm, and 495 mm. The roughness of the pipes used by Gillies was declared in the thesis, and they correspond to roughness to diameter ratios of about $3\cdot10^{-5}$. The actual roughness values were provided as input into PHOENICS to evaluate the wall shear stress of the liquid phase. Conversely, the effect of pipe roughness on the wall shear stress of the solid phase was not considered, shelving for future research the development of a suitable model to account for this effect. Among the different types of solid particles tested by Gillies, the fine silica sand with a density of 2650 kg/m^3 and a narrow particle size distribution with a mean diameter of 0.18 mm was compatible with the range of applicability of the β-σ two-fluid model. For the smaller pipe experiments, the area-averaged concentration C_{vi} ranged from 15% to 30%, while the mean flow velocity V_{m} varied from 1.5 to 3.5 m/s. For the large pipe experiments, C_{vi} ranged from 10% to 34%, and V_{m} was between 2.74 and 4.26 m/s. It's worth noting that, although most of the largest pipe cases did not satisfy the second applicability condition of the β-σ model (Eq. 6) due to a mean flow velocity lower than $1.5V_{\mathrm{dl}}^{\mathrm{T}}$=4.09 m/s, no deposition was observed in Gillies' experiments. Hence, those cases with V_{m} lower than $1.5V_{\mathrm{dl}}^{\mathrm{T}}$ were still considered.

Gillies' data regarding the 53.2 mm pipe were used for the calibration of the model. The calibration strategy described in Section 2.3 requires experimental data for two testing cases, one at a low concentration (10%-15%) and the other at a high concentration (35%-40%). However, the highest in-situ concentration considered in the experiments of Gillies was approximately 30%, which seems insufficient for an effective calibration of β. Consequently, an alternative calibration strategy was employed. Particularly, the values of β and σ were determined based on the overall agreement with the entire database of measurements from Gillies (8 tests in total), thus making, from a certain standpoint, calibration and validation indistinguishable. Starting from the ranges of β and σ identified in Messa *et al.* (2023), the following three combinations were compared, namely, β=0.10 and σ=0.50, β=0.50 and σ=0.50, and β=0.25 and σ=0.75. The results for the smallest pipe cases are presented in Figures 1 and 2. The calibration of σ, based on all measured concentration profiles, indicated that σ=0.50 was a reasonably accurate value, as shown in

Figure 1 based on two exemplary cases. Additionally, the chord-average concentration profile was found to be insensitive to changes in β, even at a high concentration of approximately 30%. Figure 2 demonstrates that β=0.10 produces slightly larger overall deviations in hydraulic gradient than β=0.50 and β=0.25, as it is evident by comparing the Mean Absolute Percentage Errors (MAPEs). Based on the above, the pair of values σ=0.50 and β=0.50 was chosen as a generally accurate representation of the available experimental data, considering both the solid concentration profile and hydraulic gradient.

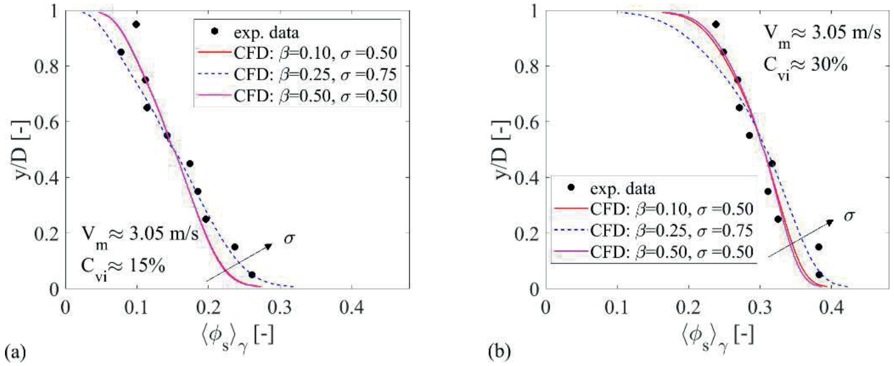

Figure 1 Calibration of the model for Gillies' tests in the small (53.2 mm) pipe; exemplary concentration profiles for different pairs of β and σ. The symbols $\langle\phi_s\rangle_\gamma$ and y/D denote the chord-average concentration and the relative elevation above the pipe bottom, respectively.

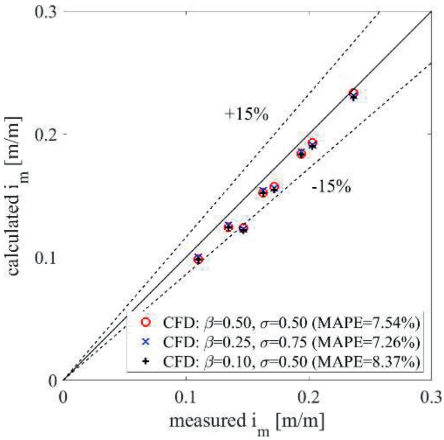

Figure 2 Calibration of the model for Gillies' tests in the small (53.2 mm) pipe; parity plot of predicted vs measured hydraulic gradient for different pairs of β and σ, with indicated the Mean Absolute Percentage Errors (MAPEs).

Subsequently, the model calibrated using the data referring to the small pipe (53.2 mm) was applied to simulate the large pipe (495 mm) tests, and the results of the validation are depicted in Figure 3. Only two representative concentration profiles, among the four that meet the applicability limits of the *β-σ* two-fluid model for the 495 mm diameter pipe, are shown here. Although the testing conditions are insufficient to reach any definitive conclusion, it appears that a 0.50 value for *σ*, obtained from the calibration on the small pipe, does not allow predicting the concentration profile for a larger pipe accurately (Figure 3a), which basically means that *σ* is dependent on the pipe diameter. At the same time, the hydraulic gradient predictions in the large pipe remain considerably satisfactory, within a ±15% deviation (the MAPE is even lower than that in the calibration phase, Figure 3b).

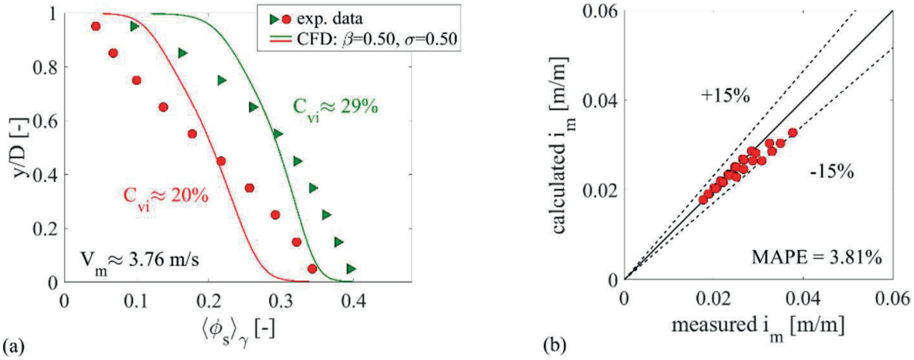

Figure 3 Validation of the model with *σ*=0.5 and *β*=0.5 for Gillies' tests in the big (495 mm) pipe: (a) exemplary comparison of concentration profiles for $V_m \approx 3.76$ m/s; (b) parity plot of predicted vs measured hydraulic gradient.

Considering that the concentration profile is primarily influenced by *σ*, while the hydraulic gradient is primarily affected by *β*, it might be reasonably argued that *β* shows good extrapolability to larger pipe diameters.

In order to further explore the effect of changing the pipe size on the appropriate *σ*, the calibration of this parameter was also performed for the two other pipe diameters tested by Gillies, namely, 159 mm and 495 mm. The value of *β* was kept as 0.50, whilst the value of *σ* was decided based on the overall agreement with respect to the measured concentration profiles fulfilling the applicability conditions of the *β-σ* two-fluid model, which were in the number of 6 for *D*=159 mm and in the number of 4 for *D*=495 mm. As it can be seen in Figures 1 and 4 for a few exemplary cases, reasonable values of *σ* were found to be 0.50, 0.75, and 1.25 for pipe diameters of 53.2 mm, 159 mm, and 495 mm, respectively. This suggests that *σ* should increase with the pipe diameter, but other data are required to obtain a mathematical correlation.

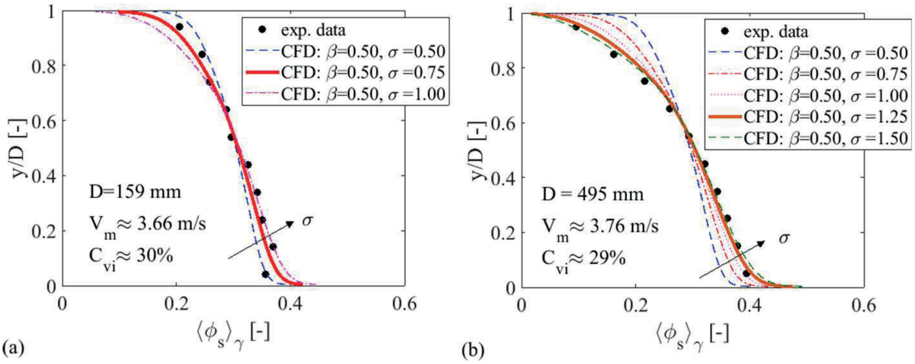

Figure 4 Exemplary results of calibration of σ for Gillies' tests in the medium (159 mm) and big (495 mm) pipes. In all cases, β was kept fixed to 0.50.

4.2. EXTRAPOLATION TO DIFFERENT PARTICLE MATERIAL

The "extrapolability" of the calibrated values of β and σ to different particle materials is also a subject of practical importance. Among the other advantages, it would simplify the execution of the calibration tests, for instance opening the possibility of running the experiments using glass beads instead of more abrasive sand-like media.

A major challenge of this study arises from the difficulties in finding data for experimental conditions differing only in the material of the solids. Indeed, Schaan *et al.* (2000) conducted an experimental study on slurry pipe flows using two different facilities and testing various particle materials, namely glass beads, Lane Mountain sand, and Ottawa sand, with broadly the same grain size of about 0.10 mm. Notwithstanding, concentration profiles were provided only for Lane Mountain sand and, therefore, it was not possible to utilize this data for the purpose of assessing the influence of particle material on β and σ.

Table 1

Test cases for the analysis of the extrapolation to different particle material. For all cases, the pipes are hydraulically smooth, V_m was approximately between 2 and 5 m/s, and C_{vi} approximately between 10 and 40%.

Case	Reference	D [mm]	d_p [mm]	Particle material	β [-]	σ [-]
A	Kaushal and Tomita (2007)	54.9	0.15	Glass beads	0.25	0.50
B	Matoušek (2002)	150.0	0.13	Sand	0.25	0.75
C	Schaan et al. (2000)	158.5	0.09	Lane Mountain sand	3.25	0.75
D	Schaan et al. (2000)	53.2	0.09	Lane Mountain sand	3.25	0.50

Thus, we had no other choice than referring to experimental data from different literature sources, bearing in mind that differences in the experimental facilities, instrumentation, and testing procedures might affect the quality of the comparison and, indeed, only indications and no definitive answers could be obtained. The four test cases that were considered for the analysis, all referring to turbulent flow, are summarized in Table 1. The table also provides the values of β and σ obtained by applying the calibration procedure to each case. Note that the calibration of cases A to C had already been reported in Messa *et al.* (2023), whilst that for case D was an original result of this paper.

The calibration of case D indicated that β=3.25 and σ=0.50 provide reasonably accurate predictions of concentration profile and hydraulic gradient, as shown in Figure 5, red data. Conversely, when simulating case D with β=0.25 and σ=0.50, which are the values obtained for case A, the predictive capacity is still good in terms of concentration profile but poor in terms of hydraulic gradient, as shown in Figure 5, blue data. Since cases A and D basically differ only in terms of particle material (indeed, also the particle size is not the same, but both values fall in the range of "fine particles"), the following considerations might be drawn. The value of σ, which mainly affects the predicted concentration profile, seems extrapolatable to different particle materials. However, this does not seem to be the case for the value of β, which appears material-dependent and has a noticeable effect on the predicted hydraulic gradient, with a detectable yet not significant influence on the predicted concentration profile. This guess is further confirmed by comparing the calibrated values for cases C and D, in which a Lane Mountain sand slurry flows in pipes with different diameters. Here σ is different, since, as already observed, this parameter depends on the pipe diameter; at the same time, β is the same. However, as already noticed, no definitive conclusion can be reached at the moment; in fact, the chosen β is the same for cases A and B, although a different material is transported in the two sets of experiments (glass beads and silica sand). Thus, it might be argued that β should be changed only for certain types of materials; indeed, the Lane Mountain one is a peculiar type of sand, characterized by a particularly significant angularity producing high frictional losses.

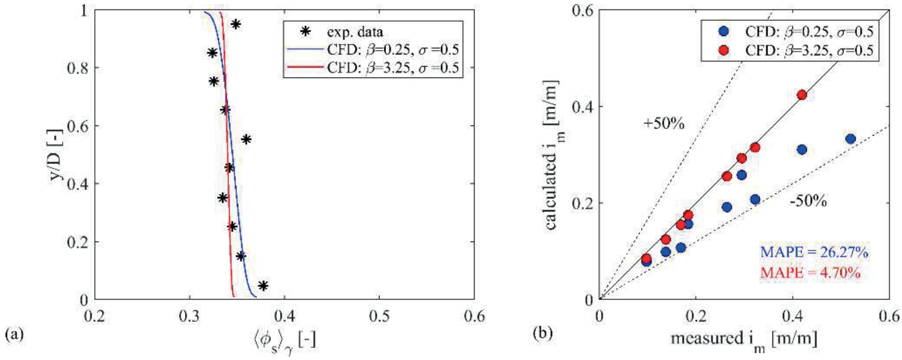

Figure 5 Validation of the model with σ=0.5 and β=0.25 for case D in Table 1 and subsequent recalibration with σ=0.5 and β=3.25 (a) exemplary comparison of concentration profiles for $V_m \approx 3$ m/s; (b) parity plot of predicted vs measured hydraulic gradient.

5. CONCLUSION

This paper arises as the continuation of a recently published article (Messa *et al.* 2023), focused on the assessment of the predictive capacity of the β-σ two-fluid model for pseudo-homogeneous slurry pipe flows. Particularly, here we focused on the "extrapolability" of the calibration coefficients, that is, the assessment of whether the calibrated values of β and σ are still appropriate when changing the pipe diameter and the particle material. Although no definitive conclusions could be reached without running dedicated experiments, some tendencies could be found when referring to experimental data taken from different literature sources. In summary, the proper value of σ, which mainly affects the concentration profile, should increase with the pipe diameter, without being much affected by the particle material. Conversely, the value of β, which mainly affects the hydraulic gradient at high concentration, with a minor influence also on the concentration profile, does not appear to be a function of a pipe diameter but it is somehow material dependent, at least for certain classes of solids. Further research is being carried out to turn this discussion into practical criteria and correlations to decide about the values of β and σ outside of the calibration conditions.

ACKNOWLEDGEMENTS

The doctoral research of Qi Yang at Politecnico di Milano was financed by China Scholarship Council (CSC).

REFERENCES

1. Cheng, N.S, Law, A.W.K., 2003. Exponential formula for computing effective viscosity. Powder Technol. 129, 156–160.
2. Gillies, R.G., 1993. Pipeline flow of coarse particle slurries. PhD Thesis, University of Saskatchewan, Canada.
3. Kaushal, D.R., Tomita, Y., 2007. Experimental investigation for near-wall lift of coarser particles in slurry pipeline using γ-ray densitometer. Powder Technol. 172, 177–187.
4. Korving, A.C., 2002. High concentrated fine-sand slurry flow in pipelines: experimental study, Proc. 15th Int. Conf. Hydrotransport, Banff, Canada, 3-5 January 2002.
5. Matoušek, V., 2002. Pressure drops and flow patterns in sand-mixture pipes, Experimental Thermal and Fluid Science, 26, 693–702.
6. Messa, G.V., Matoušek, V., 2020. Analysis and discussion of two fluid modelling of pipe flow of fully suspended slurry. Powder Technol. 270, 747-768.
7. Messa, G.V., Yang, Q., Adedeji, O.E, Chára, Z., Duarte, C.A.R., Matoušek, V., Rasteiro, M.G., Sanders, R.S., Silva, R.C., de Souza, F.J., 2021. Computational Fluid Dynamics Modelling of liquid–solid slurry flows in pipelines: State-of-the-Art and Future Perspectives. Processes 9, 1566.
8. Messa, G.V., Yang, Q., Rasteiro, M.G., Faia, P., Matoušek, V., Silva, R.C., Garcia, F., 2023. Computational Fluid Dynamic modelling of fully-suspended slurry flows in horizontal pipes with different solids concentrations. KONA Powder Particle J., 40, 219-235.
9. Schaan, J., Sumner, R.J., Gillies, R.G., Shook, C.A., 2000. The effect of particle shape on pipeline friction for Newtonian slurries of fine particles. Can. J. Chem. Eng. 78, 717-725.
10. Schiller, L., Naumann, A., 1935, A drag coefficient correlation. Z. Ver. Deutsch. Ing. 77, 318-320.
11. Spalding, D.B., 1981. Numerical computation of multi-phase fluid flow and heat transfer. In Von Karman Institution for Fluid Dynamics Numerical Computation of Multi-Phase Flows, 161–191.
12. Thomas, A., 2015. A modification of the Wilson & Judge deposit velocity equation, extending

its applicability to finer particles and larger pipe sizes. Proc. T&S Conf., Delft, The Netherlands, 22–25 September 2015.

13. Wilson, K.C., Addie, G.R., Sellgren, A., Clift, R., Slurry transport using centrifugal pumps, 3rd ed., Springer, New York, NY, USA, 2006.