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DISPERSION IN THE RESIDENCE TIME OF SIZE-DISPERSED PARTICLES IN SEDIMENTATION

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ABSTRACT: Theoretical expressions have previously obtained for the statistics of the residence time distribution of particles falling individually in a stationary, Newtonian liquid. The dispersion in the residence or sedimentation time arises both from the size dispersion that may be present in the particles and also because of fluctuations in the axial velocity of the particles about the time-invariant terminal velocity. Such fluctuations are inevitable, except at extremely low Reynolds numbers. The size dispersion is represented by the Log-Normal distribution, as is customary for many particle populations. The erratic nature of particle velocity is represented by a dispersion coefficient and then incorporated into a corresponding Peclet number. The dispersion coefficient reflects both the level of fluctuation in velocity and the representative time-scale of the velocity fluctuation. In addition to residence time distribution, the level of correlation or dependence between particle size and particle residence time can be determined by this method. The theoretical work was previously validated using glass and plastic particles falling in glycerol and water, characterized by low ($Re \approx 1$) and high ($Re \approx 1000$) Reynolds numbers, respectively. For this paper, new experiments were conducted examining the fall of expanded polystyrene particles with a range of sizes in air. Experiments were carried out with single particle falls and batch (groups of particles) falls. In addition to using different fluids and particles to the previous work, the tests were conducted over a wider range of Reynolds numbers. Results demonstrated that the theory was still valid for these new experiments. Dispersion in residence time and the relationship between particle size and its residence time were predicted with reasonably good accuracy.

KEY WORDS: Falling particles, Fluctuating velocity; Size dispersion; Peclet Number; Residence time distribution.

NOTATION

a	Drag Coefficient fitting parameter
b	Drag Coefficient fitting parameter
c_D	Drag coefficient
c_τ	Coefficient of variation in residence time
c_1	Fluid-particle constant parameter (/s)
c_2	Fluid-particle exponent parameter
D	Dispersion parameter (mm^2/s)
d	Particle diameter (mm)
H	Fall height (m)

m_d Mean diameter parameter (log-normal distribution)
m_τ Mean residence time parameter (log-normal distribution)
Pe Peclet number
Re Reynolds number
r Correlation coefficient between log residence time and log droplet diameter
s_d Standard deviation diameter parameter (log-normal distribution)
s_τ Standard deviation residence time parameter (log-normal distribution)
t Time (s)
u_n Nominal particle velocity (mm/s)
u_t Terminal particle velocity (mm/s)
ξ Correlation coefficient between residence time and droplet diameter
μ_d Mean particle diameter (mm)
μ_τ Mean residence time (s)
η_f Fluid dynamic viscosity (Pa s)
ρ_f Fluid density (kg/m^3)
ρ_p Particle density (kg/m^3)
σ_d Standard deviation in particle diameter (mm)
σ_τ Standard deviation in residence time (s)
τ_n Nominal residence time (s)

1. INTRODUCTION

The motion of particles falling under gravity through a fluid is widespread in many engineering operations. To design these operations, the residence time (the time taken to fall through a certain distance) must be known, which presupposes knowledge of the falling particle velocity. Only where a single, perfectly spherical, and smooth particle falls through an infinite body of a quiescent Newtonian fluid with completely homogeneous properties at an extremely low velocity is an exact analytical solution available to predict the constant falling velocity where hydrodynamic drag balances external gravitational forces. The presence of many factors, including multi-particle interactions, any small, non-uniformity in particle size, shape and density, any off-centre location in a flow channel all affect particle velocity, Boschan et al., (2016), Mucha et al., (2004), Yong et al., (2018). The nature of the relationship between the diameter of a particle and its residence time in the system is also of interest to the process designer. Larger particles have greater inertia and tend to fall faster but the relationship depends both on the nature of the particle size distribution and on the nature of the non-uniformity in particle motion. For any particle, an expression for a nominal terminal velocity can be obtained, which in turn implies a fixed residence time for a given particle size. The actual complexity of the interactions between the particle and the surrounding fluid causes a variability in residence time. The unsteadiness in particle velocity fundamentally depends on the disturbed flow regime that is developed in the wake of the falling particle. This causes a time-varying hydrodynamic force to be applied to the particle and the time-dependence of this force will be sensitive to the prevailing particle Reynolds Number, Fornari et al., (2016), Sakamoto & Haniu, (1990). Only a mechanistic analysis of the flow field can reveal the origin of this phenomenon, Ghosh & Stockie, (2013). This paper will not theoretically analyse this

phenomenon; rather it presents a methodology that can potentially incorporate the effect of all these phenomena on residence time using a single parameter.

In previous work by Cronin et al., (2023) theoretical expressions have been obtained to predict the statistics of the residence time of particles falling in a stationary fluid incorporating a dispersion in particle size and including the influence of complexity and uncertainty in particle motion. Additionally, the statistical relationship between residence time and particle diameter was established. These expressions were validated by detailed tests carried out using a variety of systems of glass and plastic particles falling in both glycerol and water. This paper reports on new tests, dropping polystyrene beads in the air. This experimental system is more rudimentary, but it does provide new experimental data to test and extend the theory. These new results also permit more interpretation of the previous results.

2. PARTICLE RESIDENCE TIME

Assuming that a particle very rapidly achieves its terminal velocity, u_t in the fluid, the expression for residence time, τ for a particle falling through a distance, H is

$$\tau = \frac{H}{u_t} \qquad (1)$$

The three primary forces acting on a single, non-accelerating, non-rotating, spherical particle as it falls through a stationary, Newtonian fluid are gravity, buoyancy and drag force and terminal velocity can be obtained from their balance.

$$u_t = \sqrt{\frac{4\,(\rho_p - \rho_f)\,g\,d}{3C_D\rho_f}} \qquad (2)$$

To determine the dependence of terminal velocity on particle diameter, the dependence of the drag coefficient on the Reynolds Number in the region of interest must be established. The drag coefficient can be given a simple power law dependency on particle Reynolds Number and where the accuracy of this local approximation can be adjusted to any desired level of precision by refining the Reynolds Number range under consideration in order to match the output of the more general and accurate correlations, Yaghoobi & Torabi, (2012), Michaelides, (1988).

$$C_D = \frac{a}{Re^b} \qquad \text{where} \qquad \text{Re} = \frac{\rho_f\, u_t\, d}{\eta_f} \qquad (3)$$

where a and b are power law fitting parameters, and their values depend on the Reynolds Number range of interest. Hence, terminal velocity and accordingly residence time will have a power law dependency on particle diameter.

$$\tau = \frac{H}{c_1 d^{c_2}} \qquad (4)$$

and the fluid-particle coupling parameters c_1 and c_2 can be evaluated knowing the drag coefficient fitting parameters as

$$c_1 = \left(\frac{4\,(\rho_p - \rho_f)g}{3\,a\rho_f^{1-b}\eta_f^b}\right)^{\frac{1}{2-b}} \qquad c_2 = \frac{1+b}{2-b} \qquad (5)$$

The constants in the relationship, c_1 and c_2, are fixed for a given fluid-particle system and for a given Reynolds number range; the c_2 parameter quantifies the sensitivity of residence time to diameter. The systematic trend is that as particles become larger, residence time decreases and furthermore residence time becomes less sensitive to particle diameter.

3. PROBABILISTIC RESIDENCE TIME ANALYSIS

Using equation 4, an expression for the distribution in residence time that includes both the effects of dispersion in particle size and the irregular nature of particle motion can be arrived at. Nominal residence time, τ_n can be defined as equal to the fall height, H divided by nominal velocity, u_n where u_n is the constant (time-averaged) terminal velocity of the average sized particle

$$\tau_n = \frac{H}{C_1\mu_d^{c_2}} = \frac{H}{u_n} \qquad (6)$$

It should be noted that τ_n is not the mean residence time but rather the residence time of the averaged sized particle. Regarding size dispersion, for this work the Log-Normal probability density function is chosen to represent particle diameter, d with known (from experiment) statistics of mean, μ_d and variance, σ_d^2. The two parameters m_d and s_d needed to define the distribution can be obtained from the values of mean and variance of particle diameter using:

$$m_d = ln\left[\frac{\mu_d^2}{\sqrt{\mu_d^2 + \sigma_d^2}}\right] \qquad s_d = \sqrt{ln\left[1 + \frac{\sigma_d^2}{\mu_d^2}\right]} \qquad (7)$$

As particle diameter, d is a random variable, then the corresponding residence time, τ of each different particle will be distributed. Examining equation 7, it is significant that the size dispersion parameter, s_d reflects the fractional size distribution as measured by σ_d/μ_d rather than, actual size distribution as measured by the standard deviation, σ_d. This implies that for very small sized particles, even where absolute dispersion in size is not noticeable, a large s_d value may result meaning, that size dispersion could be very significant in terms of the RTD.

A second effect that must be included is that for any given particle size, there is a range of possible residence times because of the erratic nature of particle motion. This paper will not examine the precise nature of the flow regime and the resulting hydrodynamic forces

acting on the particle. It simply assumes that the overall effect is that in the vertical direction, particle vertical velocity is decomposable into a time-invariant, deterministic component and a fluctuating component. The stationary component of velocity, u_n will be the time-averaged value of particle velocity over its fall (after the initial acceleration period at the start). The unsteady component of velocity is analysed with random process theory and assumed to be zero-mean, Gaussian with a known variance, and having some characteristic time-scale of fluctuation.

The nature of the temporal fluctuations in particle velocity will vary depending on the particular particle-fluid system under analysis and the prevailing flow regime. This approach, of considering velocity to consist of a deterministic and random component, is equivalent to the plug flow model with a moderate level of axial dispersion used in residence time theory studies, Levenspiel, (1999). The amount of dispersion or randomness in particle motion can be quantified with the particle Peclet number, *Pe* as

$$Pe = \frac{u_n d}{D} \qquad (8)$$

This number is the ratio of advective (deterministic) to diffusive (random) particle motion. For this work, the magnitude of the dispersion parameter, *D* will be approximately given as the product of the variance of the fluctuating vertical velocity times the associated time-scale of the fluctuation, Nicolai et al., (1995).

The two separate effects (dispersion in particle size and time-varying particle velocity) that introduce uncertainty in residence time can each be represented by the Log-Normal distribution (the first theoretically, the second approximately). Particle residence time, τ being the product of two Log-Normally distributed variables and a scalar constant, τ_n will remain Log-Normally distributed, and statistics such as mean residence time and standard deviation in residence time can be evaluated using

$$\mu_\tau = \tau_n exp\left[\frac{s_d^2 (c_2 + c_2^2)}{2}\right] \qquad (9)$$

$$\sigma_\tau = \tau_n \sqrt{c_2^2 s_d^2 + \frac{2}{Pe}\frac{d}{H}} = \tau_n \sqrt{c_2^2 s_d^2 + \frac{2}{u_n}\frac{D}{H}} \qquad (10)$$

Equation 10 demonstrates that dispersion in residence time is due to the sum of the effect of particle size dispersion, quantified by the $(c_2 s_d)^2$ term plus the sum of the effect due to irregular particle motion, quantified by the *(2/Pe)(d/H)* term. The analysis also permits the determination of the level of correlation between residence time and particle diameter for a given system. The correlation coefficient, ζ between residence time, τ and particle diameter, *d* can be expressed as

$$\zeta_{\tau,d} = \frac{exp[r s_d s_\tau] - 1}{\sqrt{\{exp[s_d^2] - 1\}\{exp[s_\tau^2] - 1\}}} \qquad \text{where} \quad r = \frac{-1}{\sqrt{1 + \frac{\frac{2}{Pe}\frac{d}{H}}{(c_2 s_d)^2}}} \qquad (11)$$

Equation 11 shows that as the magnitude of the fluctuations in velocity increases, lowering the Peclet number, the level of correlation between residence time and particle diameter will fall, and a decreasing proportion of the variability in residence time is due to the variability in particle size.

4. MATERIALS & METHODS

Three sets of polystyrene spherical particles were assembled. These had nominal diameters of 9 mm, 15 mm and 30 mm and are designated as P9, P15 and P30 respectively. The experiments were conducted in an air-filled chamber having a height of over 33 m, although residence time was measured over a fall height of 29.85 m to permit the particles to accelerate to terminal velocity. The air temperature was 12 °C giving a corresponding fluid density of 1.23 kg/m^3 and viscosity of 1.79 x 10^{-5} Pa s. Individual particles were dropped repeatedly through the chamber, and residence time was measured by a timing system. Additionally, groups of particles were dropped in batches to compare their statistics to those of the individual tests. Table 1 contains dimensional and density data for these beads and also gives the standard deviation in the diameter parameter, s_d for the beads. It can be seen that the magnitude of s_d is almost identical to the coefficient of variation in diameter, c_d. The absolute dispersion in diameter as measured by standard deviation tends to be larger for the larger beads. However, the effect is not pronounced, so relative dispersion in size as measured by the coefficient of variation in diameter falls as diameter increases.

Table 1

Particle properties

Property		Units	Particle System		
			P9	P15	P30
Mean Diameter	μ_d	mm	9.30	14.67	29.81
Standard Deviation Diameter	σ_d	mm	0.39	0.55	0.49
Coeff. of Variation in Diameter	c_d	%	4.19	3.75	1.64
Particle Density	ρ	kg/m^3	24.1	21.9	39.7
Standard Deviation Parameter	s_d		4.19 x 10^{-2}	3.75 x 10^{-2}	1.64 x 10^{-2}

5. RESULTS

5.1. PARTICLE VELOCITY ANALYSIS

Table 2 gives the experimentally measured terminal velocity and the corresponding Reynolds number for the three bead systems. It also gives the magnitudes of the drag coefficient power law parameters, a and b and the predicted drag coefficient, C_D found using them. The magnitudes of the particle-fluid parameters c_1 and c_2 are shown and the predicted value of terminal velocity by equation 2. Reynolds numbers are in the range from 1,200 up to 12,000. The flow regime can be expected to be highly unsteady with eddies

being detached from the particle surface and vortices being shed off and convected downstream. For these high Reynolds number tests, the c_2 values approach 0.5, demonstrating that terminal velocity has a square root dependence on particle diameter for this regime; in accord with an almost constant drag coefficient. The predicted terminal velocity is within 10 % of the measured value. Clearly, larger particles fall faster with a correspondingly higher Reynolds number.

Table 2

Particle velocity

Property	Symbol	Units	Particle System		
			P9	P15	P30
Measured Terminal Velocity	u_t	mm/s	1980	2590	5320
Reynolds Number	Re		1273	2647	11256
Drag Coefficient Parameter a	a		1.95	0.86	0.42
Drag Coefficient Parameter b	b		0.21	0.10	0
Particle Drag Coefficient	C_D		0.44	0.39	0.42
Fluid-Particle Parameter c_1	c_1	/s	53.9	33.1	30.9
Fluid-Particle Parameter c_2	c_2		0.674	0.579	0.500
Predicted Terminal Velocity	u_t	mm/s	2070	2590	4810

Table 3 gives the mean particle diameter, the corresponding nominal velocity (from equation 6), the dispersion coefficient, *D*, the corresponding Peclet Number, *Pe* (from equation 8) and the ratio of fall height to particle diameter. For our previous studies with particles in glycerol and water, the magnitude of the dispersion coefficient was obtained by video analysis to determine the magnitude and time-scale of velocity fluctuation. For the work presented here, this was not possible and the dispersion coefficient is more akin to a fitting parameter. However, in the Discussion section, its magnitude will be compared to the values used for the previous studies and interpreted in the context of literature studies. Also, for this work of this paper, a single value was used for the dispersion coefficient for each of the three populations. From the table it can be seen that larger sized particles yield higher values for the Peclet number, indicating that random velocity effects on residence time should be proportionally lower.

Table 3

Particle Peclet number

Property		Units	Particle System		
			P9	P15	P30
Mean Diameter	μ_d	mm	9.30	14.67	29.81
Nominal Velocity	u_n	mm/s	2070	2590	4810
Dispersion Coefficient	D	mm^2/s	25700	25700	25700
Peclet Number	Pe		0.72	1.48	6.17
Fall Height to Diameter Ratio	H/d		3210	2035	1001

5.2. RESIDENCE TIME STATISTICS

Table 4 gives the mean residence time, μ_τ the standard deviation in residence time, σ_τ and the coefficient of variation in residence time, c_τ as measured experimentally and predicted by the model (actual data reflects the fall height of 29.85 m). The table also gives the experimental values of the correlation coefficient between residence time and diameter together with the predicted values and the proportion of variability in residence time attributable to either size dispersion or the irregular nature of the motion as predicted by the model.

Table 4

Residence Time Statistics

	Symbol	Units	Particle System		
			P9	P15	P30
Experiment					
Mean Residence Time	μ_τ	s	15.0	11.6	5.6
Standard Deviation Residence Time	σ_τ	s	0.64	0.56	0.12
Coefficient Variation Residence Time	c_τ	%	4.26	4.85	2.10
Residence Time / Diameter Correlation	$\zeta_{\tau,d}$		- 0.71	- 0.91	- 0.74
Model					
Mean Residence Time	μ_τ	s	14.42	11.55	6.21
Standard Deviation Residence Time	σ_τ	s	0.59	0.39	0.12
Coefficient Variation Residence Time	c_τ	%	4.08	3.37	1.98
Residence Time / Diameter Correlation	$\zeta_{\tau,d}$		- 0.69	- 0.64	- 0.42
Contribution of Size Dispersion		%	47.9	41.5	17.3
Contribution of Velocity Dispersion		%	52.1	58.5	82.7

The mean and standard deviation of residence time are lower for larger sized particles. The predicted mean residence time is generally within 10 % of the experimentally measured time for the tests and reflects the validity of the deterministic model of velocity. The agreement between experiment and theory for the standard deviation in residence time is also good, with predicted variability in residence time within 30 % of the measured value. Experimental and predicted correlation coefficients are also in reasonable agreement. The theory always predicts negative correlation coefficients between residence time and diameter, as expected. The closer the correlation coefficient is to -1, the more the residence time dispersion is solely as a result of particle size dispersion. It can be seen that for these tests, there is a reasonable but not overwhelming correlation between residence time and diameter and that both dispersion in size and fluctuating velocity make a comparable contribution to residence time variability. The model does tend to under-predict the level of correlation between particle size and residence time.

Figure 1 plots the experimentally measured distribution in residence time for the 9 mm beads in frequency histogram format, while Figure 2 shows experimental scatter plots of residence time versus diameter for these beads (space limitations preclude showing similar results for the other two particle systems). Examining Figure 1, the residence time for these

beads is quite dispersed, ranging from 13 s to over 17 s with a mean value of 15 s. The distribution does not appear to be Log-Normal (as expected by theory) because of the relatively small number of experimental data points. Observing Figure 2, visually there is quite an amount of scatter present (correlation coefficient of -0.71) although the trend of larger particles having shorter residence times can be identified.

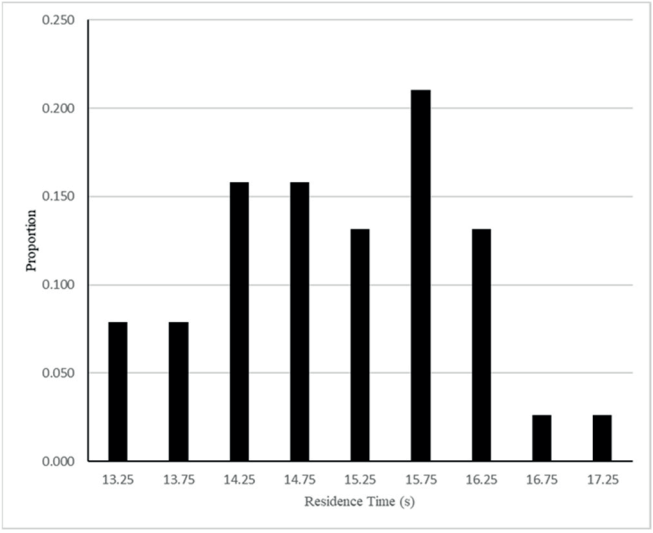

Figure 1 Experimental residence time distribution (P9 beads)

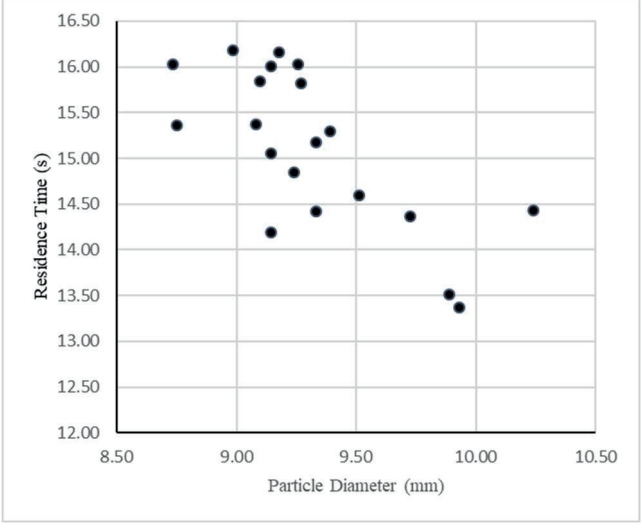

Figure 2 Experimental scatter plot of residence time and particle diameter (P9 beads)

More generally, considering all three systems of particles in terms of equation 10, it can be seen that the standard deviation in residence time consists of three terms; the nominal residence time, τ_n and then the size contribution to dispersion $(c_2 s_d)^2$ and the fluctuating velocity contribution, $(2D/u_n H)$. As the size of the particles increases from 9 mm to 15 mm and 30 mm, three general trends are present:

- Nominal residence time falls because terminal velocity increases.
- The size contribution to dispersion in residence time falls sharply because both the parameter c_2 and the size dispersion parameter, s_d fall.
- Given that the fall height, H and dispersion parameter, D are the same for all three particle systems, because terminal velocity increases with size, the contribution of fluctuating velocity to residence time dispersion also falls.

Thus, the overall standard deviation in residence time falls sharply, the coefficient of variation in residence also decreases, but less so while the ratio of the contribution of the size and velocity terms does not change significantly. Regarding the batch tests, only a brief discussion of the results is possible. As the solution was very dilute (typical inter-particle spacing was over 10 times particle diameter), the results were in accord with the single particle tests. As mean particle size increased, longitudinal dispersion diminished rapidly, which is consistent with a smaller standard deviation in residence time. Also, lateral dispersion of the cloud as it fell was an order of magnitude less than the longitudinal dispersion.

6. DISCUSSION

Variability in residence time of settling particles has been shown to be as a result of both a non-uniform particle size and velocity fluctuation effects. Using equation 10, standard deviation in residence time, σ_τ can be directly given in terms of process variables of H and u_n as

$$\sigma_\tau = \sqrt{\left(\frac{H c_2 s_d}{u_n}\right)^2 + \frac{2\,D\,H}{u_n^3}} \qquad (12)$$

Examining equation 11, the magnitude of the dispersion in residence time reflects two process parameters, including the settling velocity, u_n and the settling height, H. It can be seen that as the settling velocity increases, the standard deviation in residence time will fall continuously (all other parameters remaining constant). High nominal velocities are associated with large, dense particles falling in low viscosity liquids. The second process parameter of settling height, H is one system parameter that can be an adjustable variable (as both u_n and D tend to be intrinsic or fixed parameters). The equation demonstrates that as settling height increases, standard deviation rises and also at large settling distances, a greater proportion of the time dispersion is more as a result of non-uniformity in particle size rather than fluctuating velocity. Hence, control of size dispersion is more important in terms of keeping a fixed residence time where the settling distance is large. Finally, particle populations consisting of very small particles (sub–millimetre) with large size-dispersion settling in a viscous liquid will tend to have a very low nominal velocity, u_n while the parameters c_2 and s_d will both be relatively large. Hence, such suspensions will intrinsically

exhibit a large dispersion in the fall time. The size-dispersion component of residence time variability is proportional to the dispersion in size while the fluctuating velocity component of residence time variability is inversely related to mean particle size. Furthermore, for these small particles, as the fall distance increases, the size dispersion contribution to residence time variability becomes more important.

A key parameter of the system is the magnitude of the dispersion coefficient, *D*. Its magnitude for these tests in air can be compared to that measured for the glass and plastic particles in water and glycerol respectively to give insight into its dependence on particle properties. The data is summarised in Table 5.

Table 5

Comparison of diffusion coefficient values

Property	Units	Fluid		
		Glycerol	Water	Air
Fluid Density	kg/m^3	1260	999	1.23
Fluid Viscosity	Pa s	0.262	1×10^{-3}	1.8×10^{-5}
Representative Reynolds Number		1	2,000	5,000
Terminal (Nominal) Velocity	mm/s	40	400	3,200
Dispersion Coefficient	mm^2/s	1.2	415	25,700

The dispersion coefficient changes markedly between the three fluids, though this pattern may be specific to the flow experimental conditions studied here. Given that *D* is a function of the magnitude of any velocity fluctuation and that the size of the fluctuation (measured by standard deviation) in velocity may scale with nominal velocity [coefficients of variation ranging between 5 and 10 % are typical; Clift et al. (1978)], then faster falling particles may have larger dispersion coefficients although, it also depends on the time-scale of the fluctuations. It must be stressed that values for particle dispersion coefficient are always unique to the specific systems under study, Jourak et al., (2013), Nocentini et al. (2002), Khan et al. (2020).

7. CONCLUSIONS

A method has been presented to enable the combined effects of dispersion in particle size and the erratic nature of particle velocity on variation in particle residence time to be examined in a systematic fashion during a particle settling operation. This permits the relationship between particle diameter and residence time to be fully characterised. An example where this is important is any settling process where operations such as heat or mass transfer to the particle are occurring simultaneously. Here the outcome depends both on the contact time of the particle and the fluid, and on the particle size, and using this model, the effect of the interaction between contact time and particle size can be incorporated into the analysis. The generality of the approach means it can be applied to many different fluid-particle operations. Having a validated approach that incorporates both of these contributory factors to dispersion in residence time, means strategies to control residence time dispersion can be assessed or compared. The approach could be

extended to multi-particle systems by including a voidage function that depends on the solid/fluid loading ratio of the system.

Finally, it must be remembered that the theory of this paper was tested against the settling of large, single particles characterised by a relatively tight size dispersion and large Peclet numbers. This analysis for residence time dispersion is only valid within certain limits and is not universally applicable to all settling systems. Very highly size dispersed particles cannot be characterised using a single value for the fluid-particle coupling parameter, c_2 which imposes limits on the maximum value for s_d that can be examined. Also, the timescale for velocity fluctuation must be small compared to the residence time, which imposes a lower limit on a valid Peclet number, Pe.

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