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HOLDUP: ITS IMPORTANCE IN THE FLOW OF SETTLING PARTICLE-LADEN LIQUIDS

Trevor Jones

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TFJ Consulting Ltd.Derby,UK, DE23 3DX, info@tfjconsulting.com

ABSTRACT: The delayed and retained load of solids, *holdup*, is often overlooked as a key indicator of the characteristics of solid-liquid pipeline flow. The simplest model of the behaviour of these mixtures is the *Two-Layer Model* invented more than 50 years ago. At the time, researchers admitted that the model was far from perfect and invited refinements to improve stability and accuracy. Improvements and innovations have been applied to the holdup model described in the literature to make it useful and robust. The paper describes the holdup model, including a useful interpolation technique to predict the velocity for stationary deposition and the corresponding pressure gradient. The model shows that simply specifying a "safe" pumping velocity is a great over-simplification and suggests applications in a much wider context, for example in minimizing pipe wear.

KEY WORDS: Slurry, Holdup, Deposition velocity, Stationary Bed Locus

NOTATION

a,b,c	Quadratic coefficients $c + by - ay^2$
A,B	Coefficients in Seshadri's correlation(16)
C_v , C_r	Delivered concentration, *in situ* concentration, (*v/v*)
D,d	Duct diameter, particle diameter (*m*)
f	Fanning friction factor (-)
$\mathcal{H}$	Holdup Ratio, preferred definition $\mathcal{H} = (U - U_s)/U$
HR	Holdup Ratio, alternative definition $HR = C_r/C_v$
μ	Dynamic viscosity (*Pas*)
P , dP/dz , $\lvert dP/dz\rvert$	Pressure, pressure gradient, positive value (*Pa*, *Pa/m*)
ρ , ρ_m , ρ_s , ρ_L	density, mean, ..particle, ..liquid_ (*kg/m³*)
Re	Reynolds number (-)
sp	Relative density of particles (-)
τ_W	Wall shear stress (*Pa*)
U, U_s , U_{SBL} , U_*	Axial velocity, ..of solids, ..for a stationary bed, .. shear m/s)
v_0 , v_h	Settling velocity, hindered settling velocity (*m/s*)
(x,y)	Co-ordinates: $x = U_{SBL}$ and $y = dP/dz$
$(x_{-1}, y_{-1}), (x_0 , y_0), (x_{+1} , y_{+1})$	Extreme co-ordinates: (x_0 , y_0) being the most prominent
Z , z	Empirical exponent(15), axial displacement (*m*)

1. INTRODUCTION

Particle-liquid mixtures are often evaluated on the assumption that particles are uniformly distributed. The useful *Four-Component Model* (Wilson and Sellgren, 2001, Visintainer et al., 2017) for pump de-rating calculations is an example. Where solids have a significant settling tendency, complications arise – the requirement for a pipe velocity sufficient to avoid a stationary bed, for example. The delayed and retained load of solids, *holdup*, is often overlooked as a key indicator in these flows.

It is important to make a distinction between *settling* and *non-settling* mixtures. The *Holdup* model (and the *Two-Layer Model*) are applied to the former category. These are usually, but not exclusively, “coarse” particle mixtures. To be clear, holdup still exists in fine particle mixtures, but to a less significant extent. Other models are applicable when gravitational potentials are low. The paper demonstrates the boundary between these behaviours.

The prediction of *holdup* in settling solid-liquid mixtures involves a daunting combination of flow properties: bore diameter, concentration, diameter and density of solids, pipe velocity, internal pipe roughness, viscosity of the liquid vehicle, packing concentration, and the inclination of the pipe. An elegant nomogram was published by Professor K.C. Wilson (1979) to predict safe velocities for slurry pumping. It involved a reduction of all these factors to just three. Unfortunately, the assessment took no account of concentration, a vital factor, but later work (Wilson K.C. *et al*., 2011) included it. Luckily, over the years, a large accumulation of data has provided a very useful foundation upon which to test the output of the holdup model.

There are comparatively few references to the holdup of solid particles in liquid, although there are a greater number in solid-gas mixtures and fluidization studies. The simplest model of the behaviour of these mixtures is the *Two-Layer Model* (Wilson K.C. *et al*., 2011, Wilson K.C., 1970, Wilson K.C., 1976) invented more than 50 years ago. The underlying principle is that the flow of particle-bearing liquids in a horizontal duct can be separated into an upper suspension relying only on hydrodynamic forces and a lower layer taking support from hindered settling and the pipe walls. It has continued to evolve, and an estimate of *Holdup Ratio* is an important starting point (and ending point) in its recent spreadsheet version, *2LM* (Jones T.F., 2011, 2023). It is holdup that is the important factor, however the settling fraction is disposed (Jones T.F., 2014).

2. HOLDUP

The *Holdup Ratio* ($\mathcal{H}$) is defined here as the relative delay of the solids fraction, *i.e.*

$$\mathcal{H} = \frac{U-U_S}{U} \qquad [1]$$

where U is the pipe velocity (total throughput/cross-section area) and U_S is the velocity of the particulate solids. To this author, the crucially important part of this definition is the numerator, which is also the relative velocity of the solids flow. When solid particles approach the same velocity as the body of fluid, holdup approaches zero. At the other extreme, when solid particles have a very slow velocity with respect to the body of the flow, the holdup ratio approaches unity. It has a very powerful impact on the evaluation of particle-liquid flow.

For continuity of mass flow of solids in a pipe of diameter *D*, from a cross-section (*in situ* concentration C_r) to the delivery plane (efflux concentration C_v), the following equation must apply.

$$C_v\left(U\frac{\pi D^2}{4}\right) = U_s\left(C_r\frac{\pi D^2}{4}\right)$$

$$i.e.\ \ \frac{C_v}{C_r} = \frac{U_s}{U}$$

In [1] $\mathcal{H} = 1 - \frac{C_v}{C_r}$ [2]

Some researchers (*e.g.* Pirie,R.L. et al., 1988, Prandtl,L., 1925), define holdup ratio in a different way, as the ratio of the *in-situ* concentration to the efflux concentration $\frac{C_r}{C_v}$. There is a trap for the unwary here: this definition is algebraically connected to equations [1] and [2], but the connection between the two definitions is non-linear (see Figure 1). Claims of the comparative accuracy of models should be set alongside the definition used. The second definition has uses in measurements of very small values of holdup (as explained later) and can be converted to the preferred definition using equation [2].

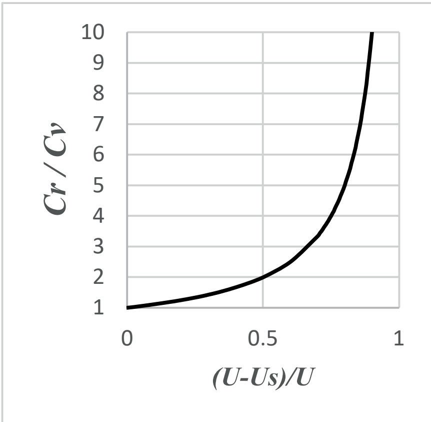

Figure 1

Figure 1: Two definitions of holdup

The velocity of the solid particles can be determined experimentally in various ways. The Holdup Ratio can be directly inferred from *γ*-ray absorption across pipe cross-sections. The "salt injection" method has also been successfully applied (Spells K.E. 1955, Pirie et al., 1988). Salt solution is injected into the duct, and the time between detection at spaced electrodes is measured. This indicates the liquid-only velocity, from which the deficit in the solids velocity can be deduced. When values are low, measurement of holdup ratio by experimental methods is very difficult, whatever method is used.

There are analytical numerical methods to predict the holdup from the intrinsic properties of a slurry.

1. Lahiri and Ghanta (2008) have an artificial neural network model, which they claim estimates the holdup with an absolute accuracy of 2.5%. The accuracy is enhanced considerably by the alternative definition of the Holdup Ratio

described above. Most of their determinations of $\frac{C_r}{C_v}$. are less than 2. Figure 1 shows that a small error on this axis is considerably magnified when referred to the horizontal axis (the $\frac{U-U_s}{U}$ definition). The accuracy achieved is impressive nonetheless. To train and test a neural network, a fairly large dataset is required. This dataset is useful in testing other analytical methods.

2. The hindered settling velocity (v_h) obtained from Richardson and Zaki (1954) can be used in a correlation for the Holdup Ratio as explained below.

Two methods are available to calculate the Holdup Ratio from the hindered settling velocity. A method by Jones (2023) uses the ratio of the hindered settling velocity to the pipe velocity to obtain

$$\mathcal{H} = 3.1179\left(\frac{v_h}{U}\right)$$

$$\text{or } \mathcal{H} = \frac{3.1179}{U} \times v_0(1 + C_v)^z \qquad [3]$$

Details of the calculation of settling velocity, v_0, *hindered* settling velocity, v_h, and empirical constant, *Z* are explained in Appendix A. The fit of equation [3] is shown in Figure 2.

The Seshadri method (Seshadri V. et al., 2001) has been applied to mixtures with very fine particles such as zinc tailings, iron ore slimes, Bauxite fines and fine coal. These are not really *settling* mixtures and provide a challenge for the holdup model. Coal particles are *hydrophobic* and may present as settling solids at finer sizes. The method employs the ratio of hindered settling velocity v_h to *shear velocity* U_*. The shear velocity arises from Prandtl's concept of *mixing length* (Prandtl,L., 1925, 1933), the distance a fluid particle travels before assimilation by the body of fluid. If there is a strong velocity gradient, components of the flow (and solid particles) exchange momentum. The idea is that an *eddy viscosity* can be defined for eddy transport in much the same way as kinematic viscosity is defined by Newton's Law of Viscosity. In regions of large velocity changes, the interface region in the *Two-Layer Model* or near the pipe walls, velocity fluctuations from turbulent eddies have the same order of magnitude as this shear velocity. The velocity gradient $\frac{dU}{dy}$ is directly proportional to U_* and inversely proportional to the mixing length. The shear velocity is simply a group of variables with dimensions of velocity. It is defined for pipe flow as follows

$$U_* = \sqrt{\frac{\tau_W}{\rho_m}} \qquad [4]$$

where τ_w =wall shear stress and ρ_m = mean mixture density across the pipe section.

$$\text{Since } \tau_W = \frac{1}{2} f \rho_m U^2 \text{ we have } U_* = U\sqrt{\frac{f}{2}} \qquad [5]$$

where f - the Fanning friction factor (*e.g.* Moody Chart (Moody L.F., 1944) or the equation of Churchill, S.W., (1977) for f).

Since $\tau_w = \frac{D}{4}\left|\frac{dP}{dz}\right|$, another form of equation [6] is

$$U_* = \sqrt{\frac{\left|\frac{dP}{dz}\right| D}{4\rho_m}} \qquad [6]$$

If needed, the following can be applied to obtain the mean density, ρ_m , of the mixture.

$$\rho_m = \rho_L\{1 + C_r(sp - 1)\} \qquad [7]$$

Where sp =the relative density of the particles and ρ_L = tdensity of the liquid vehicle.

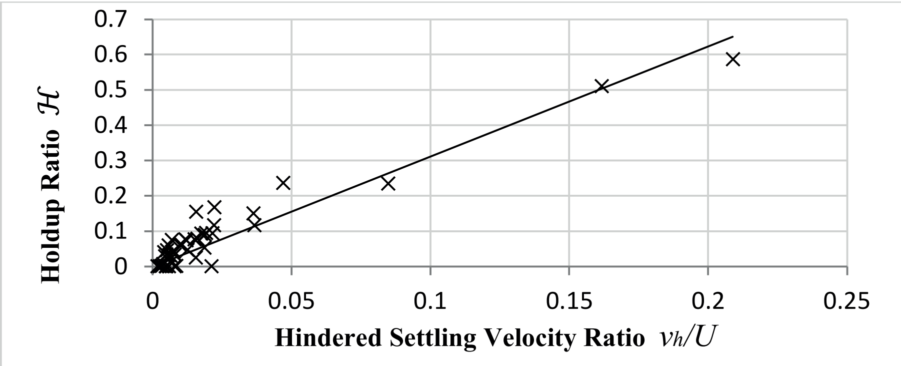

Figure2(a) Holdup Ratio as a function of v_h/U (Jones)

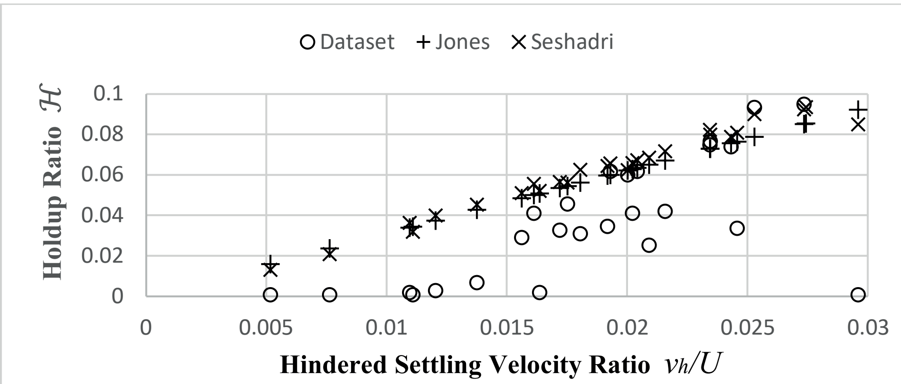

Figure 2(b) Scatter at very low values of holdup

Figure 2 Holdup Ratio as a function of hindered settling velocity ratio

From [5], the shear velocity is approximately proportional to the pipe velocity, bearing in mind the dependence of the Fanning friction factor on particle size. It has a magnitude between 5% and 10% of pipe velocity. The two approaches can be plotted to the same abscissa, v_h/U , as shown in Figure 2(b). Seshadri proposes a relation based on the alternative definition of holdup ratio (HR) as

$$HR = \frac{1}{1-\mathcal{H}} = 1 + A\left(\frac{v_h}{U_*}\right)^B \qquad [8]$$

The relation can be modified to the preferred holdup definition as follows

$$\mathcal{H} = \frac{A\left(\frac{v_h}{U_*}\right)^B}{1+A\left(\frac{v_h}{U_*}\right)^B} \qquad [9]$$

Putting $A = 0.218\ and\ B = 1.2653$ there is little to choose between the linear fit in equation [3] and Seshadri's correlation in equation [9]. At very small values of holdup there is a preponderance of zero or near zero values not gathered by the Jones or Seshadri equations, as shown by Figure 2(b). The difficulty in making the original measurements could be a contributory factor.

3. THE LIMIT OF STATIONARY DEPOSITION

Figure 3 shows the loci of points at which the model has identified a stationary bed for an example of sand in water. Note from equation [1] the holdup ratio, $\mathcal{H}$, gives the *relative velocity* of the solids. So, the hypothetical curve $\mathcal{H} = 0$ refers to the particles entrained in the flow with velocities approaching the pipe velocity, U. When $\mathcal{H} = 1$ the particles have a velocity of zero, and are entirely supported by the liquid. Both extremes are theoretical rather than practical possibilities. To the right of the envelope, there is no layered pattern, so the greater pipe velocity must accelerate slow moving particles and entrain them in the flow. The rightmost point of the curve yields the limit of stationary deposition.

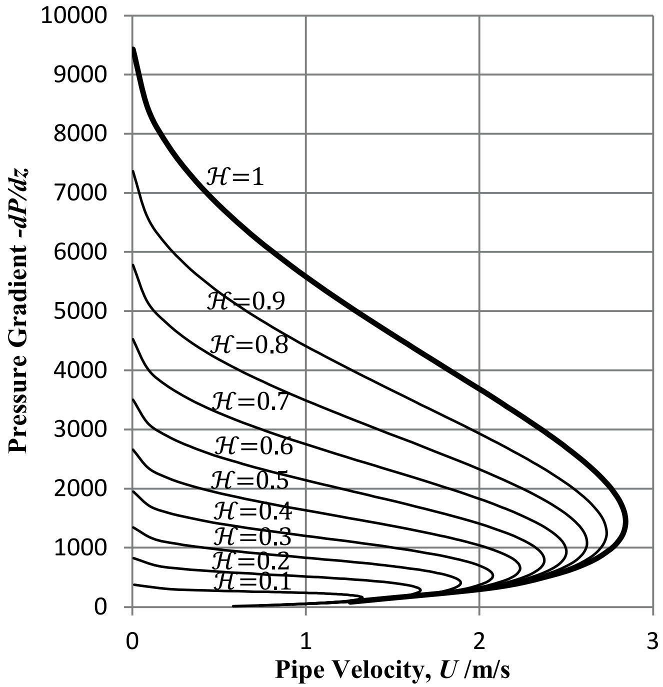

Figure 3 Stationary Bed Loci for the horizontal flow of sand in water. The concentration is 0.2 v/v, particle size is 2mm and pipe diameter 250 mm

The actual maximum velocity for a given value of $\mathcal{H}$ cannot be determined analytically. Each point is the result of a system of estimations and calculations, some of them iterative in nature (Jones T., 2023). In general, the maximum will lie between coordinates however closely they might be spaced. Interpolation of the maximum can be achieved by threading a second-order polynomial through the three most salient points as, detailed in Appendix B and shown in Figure 4 for two holdup loci: $\mathcal{H} = 1$ and 0.5.

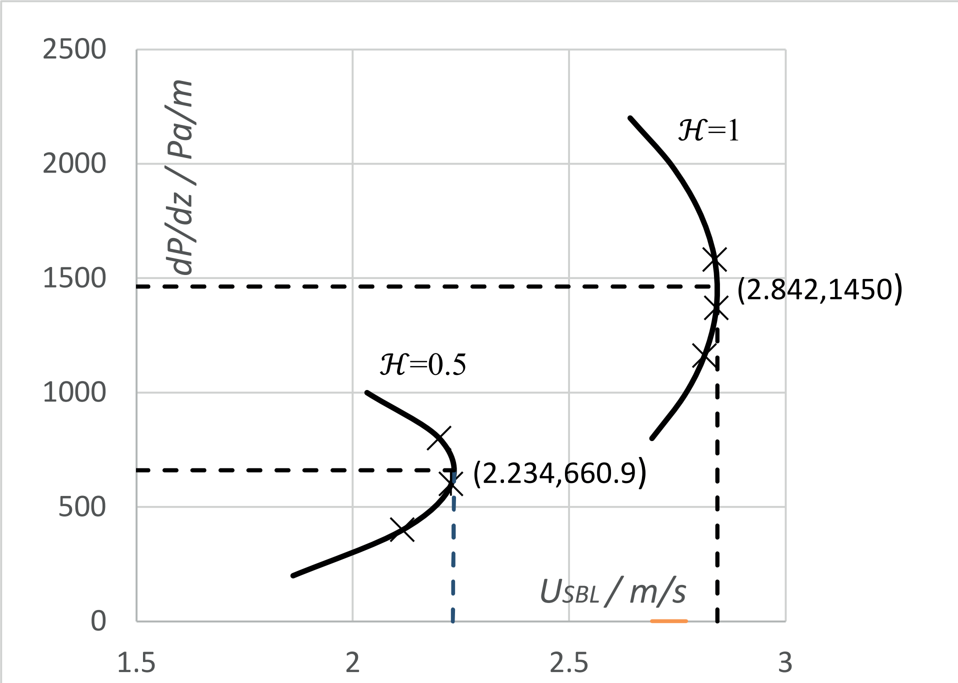

Figure 4 Interpolation of Maximum Velocity for Stationary Deposition for $\mathcal{H}=1$ and 0.5 (From Figure 3 and Appendix B)

This simple algebraic expedient is only applicable because the results are not subject to experimental uncertainty and are closely spaced. In the illustrated example, the actual discrepancy between the interpolated maximum for the $\mathcal{H} = 1$ locus (2.842 m/s,1450 Pa/m) and the most prominent result (2.840 m/s, 1371 Pa/m) is relatively small, but other outcomes, particularly at low values of holdup $\mathcal{H}$, can be quite significant. Other, more complex interpolation regimes are available in the literature (14). In Appendix B, points on the loci are expressed as coordinates (x, y), where $x = U_{SBL}$ and $y = \frac{dP}{dz}$. Notice that the second-order interpolation only applies between the three most salient points of the characteristic, $(x_{-1}, y_{-1}), (x_0\ , y_0), (x_{+1}\ , y_{+1})$ where $(x_0\ , y_0)$ is the most prominent point.

4. DISCUSSION AND RECOMMENDATIONS

The focus on a single maximum velocity for stationary deposition is clearly an oversimplification: the maximum "safe" velocity depends on the holdup ratio. Plant designers are naturally cautious and should not be blamed for specifying a velocity beyond the $\mathcal{H} = 1$ locus. Equation [1] specifies the *relative velocity* of solid particles, so the $\mathcal{H} = 1$ locus indicates a flow of liquid with zero particle velocity relative to the pipe velocity. Away from the $\mathcal{H} = 1$ envelope, high pressure, low, medium and high velocity applications are possible - for pumping shotcrete in underground tunnels, for example (Chen et al., 2016).

The high velocity required to avoid a stationary bed can be extremely damaging to economic plant operation. The pumping power required is approximately proportional to pipe velocity, raised to the power of 3. The wear propensity is approximately proportional to pipe velocity, raised to the power of 2.5. Exploratory calculations can be carried out on the model results from Figure 4 to illustrate the powerful influence of the holdup ratio. At $\mathcal{H} = 0.5$, the required axial velocity is reduced from 2.842 to 2.234 m/s at a pressure gradient reduced from 1450 to 660.9 Pa/m. This represents a significant saving in pumping power and velocity-induced wear by 45%.

The position of the settling layer, and the area of the pipe wall over which it occurs, are outputs of the model and have potential for the prediction of wear zones. The computation of the *Centre of Concentration* provides a summary indication of the position of the retained load (Jones T.F., 2023).

There are difficulties in making small holdup measurements. A small gathering of particles at the base of a duct can easily be misinterpreted, particularly their mean velocity downstream. The alternative definition of holdup becomes important in this situation. Measurements of $\frac{C_r}{C_v}$ by physical means can be converted to the preferred definition, $\mathcal{H}$, algebraically, but this begs the question as to how such physical measurements can be made. There have been historical attempts, as already flagged in this paper. The application of advanced tomographic and/or imaging techniques should improve accuracy considerably in situations where the particle density is uniform. Such experimentation might include careful calibration of the pressure gradient in particle settling conditions. Investigations of this nature are recommended if the subject of holdup is to advance.

A way to avoid excessive losses is to allow pumping at much lower values of holdup. The fitting of swirl-inducing ducts at strategic positions (Jones T.F., 2019) is a way to achieve this. In long runs of straight pipe, swirl ducts can be inserted at intervals, as in the patent from Yuille (1928). The modification is not entirely without cost however: efficient 0.3m swirl ducts suffer losses of typically 500-600 Pa (D=50mm, U=2m/s).

5. CONCLUSIONS

For settling particle-bearing liquids, the holdup model has been shown to be consistent with a comprehensive set of data. The concept of a “maximum velocity at the limit of stationary deposition” took hold in the 1950s following the work of Durand and Condiolis (1952), but of course the principle of a “safe” pumping velocity had been applied long before that decade. The painstaking work of Nora Stanton Blatch (later Blatch-Barney) on the water filtration system in Washington DC, published in 1906 (Blatch,N.S., 1906) was almost fifty years its senior. She was the first woman in the United States to obtain a degree in Civil Engineering.

Holdup ratio has been shown to be a very important variable, and methods to estimate it from desk calculations have been demonstrated using data from a wide selection of sources. The work by Professor Seshadri and his co-authors applied to fine particles differed in the values of coefficients A and B when applied to the dataset of somewhat larger particles gathered by Lahiri and Ghanta among others. It was very interesting to compare the correlation using the shear velocity with that using pipe velocity. Seshadri,V. *et al* (2001) claimed that the correlation using $\frac{v_h}{U_*}$ values below 0.17 gave a fit within 1% for 80% of the data. In the dataset applied here, only two measurements out of 44 were below 0.17.

APPENDIX A: Calculation of Hindered Settling Velocity

The following standard equations are for the settling velocity (v_0) at different Reynolds numbers (*Re*):

$$v_0 = \left(\frac{g}{18\mu}\right)(\rho_s - \rho_L)d^2 \qquad Re < 1 \text{ (Stokes Law)} \qquad \text{[A1]}$$

$$v_0 = \frac{0.2d^{1.18}\left(\frac{\rho_S-\rho_L}{\rho_L}\right)^{0.72}}{(\mu/\rho_L)^{0.45}} \qquad 1 < Re < 1000 \text{ (Allen's Law)} \qquad \text{[A2]}$$

$$v_0 = 1.74\sqrt{gd\left(\frac{\rho_S-\rho_L}{\rho_L}\right)} \qquad Re > 800 \text{ (Newton's Law)} \qquad \text{[A3]}$$

The correlation for hindered settling from Richardson and Zaki (1954) is

$$v_h = v_0(1 + C_v)^z \qquad \text{[A4]}$$

The following empirical constants were used:

$$Z = 4.65 + 1.95\left(\frac{d}{D}\right) \qquad 0.002 < Re \leq 0.2 \qquad \text{[A5]}$$

$$Z = \left(4.35 + 17.5\left(\frac{d}{D}\right)\right)Re^{-0.03} \qquad 0.2 < Re \leq 1.0 \qquad \text{[A6]}$$

$$Z = \left(4.45 + 18\left(\frac{d}{D}\right)\right)Re^{-0.1} \qquad 1 < Re \qquad \text{[A7]}$$

Piece-wise combination of equations for *Z* give rise to discontinuities at the breakpoints. Some researchers add breakpoints at $Re = 0.1$ and $Re = 400$ or use a single function to give smoother transitions. In the context of this paper, estimation of settling velocity is an initial value to a refining process using the *Two-Layer Model* (Jones T., 2023). Small discontinuities are relatively unimportant.

APPENDIX B: Polynomial Estimation of Deposition Limits

The three most significant values of $\left(U_{SBL}, \frac{dP}{dz}\right)$ in the deposition loci (Figure 3) can be joined together in a single second order polynomial with coefficients *a,b,c*. The objective is to evaluate the maximum point $\left((U_{SBL})_{limit}\ ,\ \left(\frac{dP}{dz}\right)_{limit}\right)$.

Putting $U_{SBL} = x$ and $\frac{dP}{dz} = y$ for convenience

$$x_{-1} = c + by_{-1} - ay_{-1}^2 \qquad \text{[B1]}$$

$$x_0 = c + by_0 - ay_0^2 \text{ , the most prominent value of } x \qquad \text{[B2]}$$

$$x_{+1} = c + by_{+1} - ay_{+1}^2 \qquad \text{[B3]}$$

Eliminating *c* by subtraction [B1]-[B2], and dividing by $(y_{-1} - y_0)$

$$\frac{x_{-1} - x_0}{(y_{-1} - y_0)} = b - a(y_{-1} + y_0) \qquad \text{[B4]}$$

Similarly

$$\frac{x_0 - x_{+1}}{(y_0 - y_{+1})} = b - a(y_0 + y_{+1}) \qquad \text{[B5]}$$

From [B4] and [B5]

$$a = \frac{\left[\frac{x_0 - x_{+1}}{(y_0 - y_{+1})} - \frac{x_{-1} - x_0}{(y_{-1} - y_0)}\right]}{y_{-1} - y_{+1}} \qquad \text{[B6]}$$

$$b = \frac{x_0 - x_{+1}}{(y_0 - y_{+1})} + a(y_0 + y_{+1}) \qquad \text{[B7]}$$

$$c = x_0 - by_0 + ay_0^2 \qquad \text{[B8]}$$

For the extremum $\frac{dx}{dy} = 0,\ i.e.\ b - 2ay = 0$ [B9]

Hence

$$\boxed{\begin{gathered}\left(\frac{dP}{dz}\right)_{limit} = \frac{b}{2a} \\ U_{SBL\ limit} = c + b\left(\frac{dP}{dz}\right)_{limit} - a\left\{\left(\frac{dP}{dz}\right)_{limit}\right\}^2\end{gathered}}$$

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