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DETERMINATION OF THE CRITICAL REYNOLDS NUMBER IN A TRANSITIONAL FLOW OF LIME SLURRY WITH THE ADDITION OF DEFLOCCULANT

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ABSTRACT: The article deals with the transportation of lime slurry from the reservoir to a settler in a chosen industrial enterprise. The slurry is non-Newtonian and viscoplastic. Such a slurry emphasises high viscosity and is difficult to transport under a high solid concentration in a pipeline. The amount of specially designed deflocculant was developed on the basis of rheological measurements. The addition of deflocculant allowed the reduction of friction in the pipeline and, as a consequence, allowed transportation with a higher solid concentration. This resulted in a decrease in the power consumption of the pump motor and significant energy savings in the slurry transport process.

The paper presents the determination of the parameters of the transition from laminar to turbulent flow of lime slurry with the addition of a certain amount of deflocculant. The purpose of the investigation was to determine the critical value of the Reynolds number in the slurry flow with a volume concentration of 18.28%, 23.70% and 29.36% to determine the optimal flow conditions.

KEY WORDS: lime slurry; critical Reynolds number; friction factor; laminar-turbulent transition; deflocculant

NOMENCLATURE

a	parameter calculated from Equation (12)
C_m	mass concentration of slurry (%)
C_v	volume concentration of slurry (%)
D	pipe diameter (m)
DFL	deflocculant
F	friction factor for Bingham plastic fluids
f_L	friction factor for laminar flow
f_T	friction factor for turbulent flow
m	parameter calculated from Equation (14)
N_{He}	Hedström number
N_{Re}	Reynolds number
$(N_{Re})_{cr}$	critical value of the Reynolds number

U_s	bulk velocity (m/s)
z_i	distance from datum level (i=1, 2)
Δh	difference in level between the pump inlet and the pipeline outlet (m)
Δp	pressure drops (Pa)
Δp_{1-2}	pressure drop as a result of friction (Pa)
η_p	plastic viscosity (Pa·s)
ρ_m	density of slurry (kg/m^3)
τ	wall shear stress (Pa)
τ_y	yield stress (Pa)
γ_1	specific gravity of the slurry in cross section 1-1 (N/m^3)
γ_2	specific gravity of the slurry in cross section 2-2 (N/m^3)
$\dot{\gamma}$	shear rate (s^{-1})
Φ_c	parameter calculated from Equation (8)

1. INTRODUCTION

Pipeline transport is one of the most reliable and economical modes of moving a solid phase over a long distance. In addition, it has numerous ecological values. The transportation of a slurry with a high solid concentration requires the determination of rheological properties on the basis of viscometric measurements in the form of flow curves. One of the significant problems encountered in the flow of slurry is determining the transition from laminar to turbulent motion that occurs in a certain zone of flow velocity for specific slurry properties. The transition from laminar to turbulent flow is described by the critical value of the Reynolds number, below which the tested slurry moves in laminar motion, and after exceeding it in turbulent motion. Transport in turbulent motion is characterised by a significantly higher energy demand by the pump than in the laminar motion. The friction forces that occur inside the slurry itself and in the vicinity of the pipe walls influence the losses measured by the energy consumption during pumping of a slurry and the failure rate of the devices transporting the slurry.

Fine particles enhance the viscosity of the slurry, and this may be expected to delay the transition to turbulent flow. Particles may also modify the velocity profile and thereby influence the flow transition too (Chhabra and Richardson, 1999), (Matas et al., 2003). Senapati et al. (2009) investigated the rheological behaviour of limestone-water slurry samples at different solids volume concentrations, particle size distributions, and slurry temperatures. The relative viscosity of the slurry (the ratio of slurry viscosity to suspending liquid viscosity) was found to decrease with increasing temperature in the temperature range studied (30°C to 50°C).

Difficulties arise if the slurry has a significant yield stress. In such a case, the transition from laminar to turbulent flow may be encountered at velocities of commercial interest with pipes of a wide range of diameters. This contrasts with the 'settling slurries' in which the flow is almost always turbulent. The analysis of the appearance of the yield stress in slurries was performed by Michaels and Bolger (1962). The authors reasoned that the Bingham yield stress has two components, i.e. the true network strength, which must be overcome for motion to occur at all, and a creep energy dissipation effect accompanying the collisions between flocs. The phenomenon of increasing the viscosity at the wall was indirectly confirmed by Squires and Eaton (1990), who used direct numerical simulation of isotropic turbulence to investigate the effect of turbulence on the solid concentration of heavy particles. Some other researchers reasoned that a laminarisation effect in fine dispersive slurries exists (Wilson and Thomas, 1985), (Bartosik, 2008). In conclusion, slurry flow with fine solid particles is very complex, and it remains a challenge to perform experiments on the influence of the solid particles on the transition to turbulence.

The purpose of the research was to determine parameters of the transition from laminar to turbulent flow of a slurry consisting of fine particles of limestone and water, such as the critical Reynolds number and the maximum permissible velocity for laminar flow conditions. Furthermore, the friction factor was determined.

The volume concentration range of the solid phase in the mixture was 18.28%, 23.70% and 29.36%, which corresponds to its weight concentration of 35.00%, 42.75% and 50.00%.

2. TEST MATERIAL AND MEASUREMENT METHOD

The lime slurry used in the research came from one of the Polish limestone mines. The slurry is transported in the final stage of lime production. The solid fraction of the slurry consists of fine limestone particles containing a high percentage of calcium carbonate. The grain composition of the lime slurry was determined using a laser grain size analyser. The grain size of the solid lime particles ranged from 0.50 μm to 163.50 μm, with an average grain size of 45.5 μm. The largest share in the sample had the dust fraction (65%) with a grain diameter in the range of 2-50 microns. The sand fraction with grain diameter greater than 50 microns represented almost a third of all particles. The remaining particles with grain diameters smaller than 2 microns belonged to the clay fraction. Chemically, lime solids consist mainly of calcium oxide (74%) and silicon oxide (13%). Other chemical compounds contained in the lime slurry are magnesium oxide, iron oxide, aluminium oxide, and sulphur trioxide.

Experimental studies on rheological properties were carried out in the range of slurry density from 1255 kg/m^3 to 1410 kg/m^3, which corresponds to the volume concentration of the solid phase C_v=(18.28−29.36) % and the mass concentration C_m = (35−50) %. The density of the carrier liquid, which was water, was 998.2 kg/m^3, while the density of the solid phase was 2400 kg/m^3. Rheological measurements of slurries at a temperature of 20ºC were performed using a rotational viscometer. The rheological properties were determined using the Anton Paar MCR 302 rotational rheometer. Flow curve measurements in slurry samples were made, that is, the dependence of the shear stress (τ) on the shear rate ($\dot{\gamma}$). Analysis of the experimentally obtained flow curves allowed us to conclude that the tested slurries were non-Newtonian, because their viscosity $(\eta_p = \tau/\dot{\gamma})$ decreased with increasing shear rate ($\dot{\gamma}$). Furthermore, the extrapolation of all the flow curves of the slurries to value $\dot{\gamma} = 0$ indicates the existence of a yield stress, i.e., $\tau_y > 0$ for $\dot{\gamma} = 0$. Slurries with such flow curves belong to the group of non-Newtonian viscoplastic liquids or shear thinning liquids with yield stress (Macosco, 1994), (Dziubiński et al., 2014), (Mezger, 2014).

The slurry flow analysis was carried out for a physical model of a pipeline with a constant diameter of 200 mm and a total length of 632 m.

3. RESULTS OF EXPERIMENTS AND PREDICTIONS

The study presents a method for determining the transition from laminar to turbulent flow by defining the critical value of the Reynolds number for lime slurries with the addition of a deflocculant consisting of sodium-water glass and calcareous groats. Groat is a post-production waste that comes from the lime slaking process. The optimum amount of deflocculant was set on the basis of rheological measurements in which the effectiveness criterion was a minimum viscosity value.

The article presents the results of rheological tests for slurries with volume concentrations of 18.28%, 23.70% and 29.36% without an additive (marked as ‘pure’) and with the addition of a deflocculant (marked as ‘+ DFL’). The estimated range of the shear rate that occurs in the flow of the analysed pipeline is (2÷70) s^{-1}.

Measurements indicated that the tested slurries are non-Newtonian liquids. Viscoplastic liquids combine the rheological properties of viscous liquid and solids, which is manifested by the presence of a flow limit (Dziubiński et al., 2014).

The rheological properties of the slurries tested in the studied range of the shear rate were described with high accuracy by the Bingham plastic model as follows:

$$\tau = \tau_y + \eta_p \cdot \dot{\gamma} \tag{1}$$

where η_p is the plastic viscosity and τ_y is a yield stress.

The flow curves of the tested slurries compared with the results calculated for the Bingham plastics model (solid line) are presented in Figure 1.

In Figure 1 shows a trend of decreasing the viscosity of the slurry after the addition of deflocculant. This decrease in viscosity occurs with an increase in the volume concentration of the slurry. The highest value of the decrease in the viscosity of the slurry was achieved for the slurry with the highest concentration value of the volume tested.

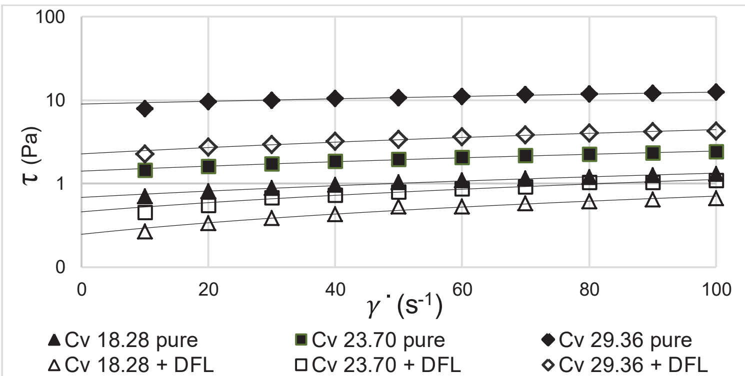

Figure 1 The experimental flow curves of tested slurries.

The characteristic parameters determined that describe the Bingham plastic slurry with the addition of a deflocculant, such as the density of the slurry (ρ_m), yield stress (τ_y), plastic viscosity (η_p) and the coefficient value of fitting the mathematical model to the experimental data (R^2) are presented in Table 1.

Table 1

The rheological parameters of the slurries tested with the addition of deflocculant

C_v (%)	ρ_m (kg/m^3)	τ_y (Pa)	η_{pl} (Pa·s)	R^2 (%)
18.28	1254	0.2461	0.0046	98.62
23.70	1330	0.4576	0.0066	98.55
29.36	1410	2.2654	0.0216	97.85

Taking into account the slurry with the optimum amount of deflocculant, which was 0.5% in relation to the mass of the solid phase in the slurry and the proportion of components of 1:1, the mathematical model was formulated to calculate the friction in a pipeline. To formulate the mathematical model, the physical model of slurry flow must first be set up.

The fine-dispersive slurry is assumed to be transported by centrifugal pump from the reservoir to the natural settler in a pipeline with an inner diameter of 200 mm. The total length of the pipeline is 632 m. The inner diameters of the suction and discharge pipes are the same. The total height difference between the inlet and the outlet length of the pipeline is 11 m. The pipeline scheme with chosen cross sections 1-1 and 2-2 is presented in Figure 2.

The general form of the Bernoulli equation for chosen cross sections 1-1 and 2-2 and datum level 0-0 is the following:

$$\frac{\rho_1 \cdot U_{S1}^2}{2} + p_1 + \gamma_1 \cdot z_1 + \Delta p = \frac{\rho_2 \cdot U_{S2}^2}{2} + p_2 + \gamma_2 \cdot z_2 + \sum \Delta p_{1-2} \tag{2}$$

Taking into account the assumptions made in the physical model, Equation (2) can be written as:

$$0 + p_{atm} + \gamma_m \cdot (h_0 + h_1) + \Delta p = \frac{\rho_m \cdot U_S^2}{2} + p_{atm} + \gamma_m \cdot z_2 + \sum \Delta p_{1-2} \tag{3}$$

Th

$$\Delta \mathrm{p} = \frac{\rho_m \cdot U_s^2}{2} + \rho_m \cdot g \cdot \Delta h + \Delta p_{1-2} \tag{4}$$

where ρ_m is the density of the slurry, U_s is the bulk velocity, and $\Delta h = z_2 - h_0 - h_1$ is the difference in level between the pump inlet and the outlet of the pipeline. The Δp_{1-2} represents pressure losses due to friction, which were calculated from the Darcy–Weisbach equation.

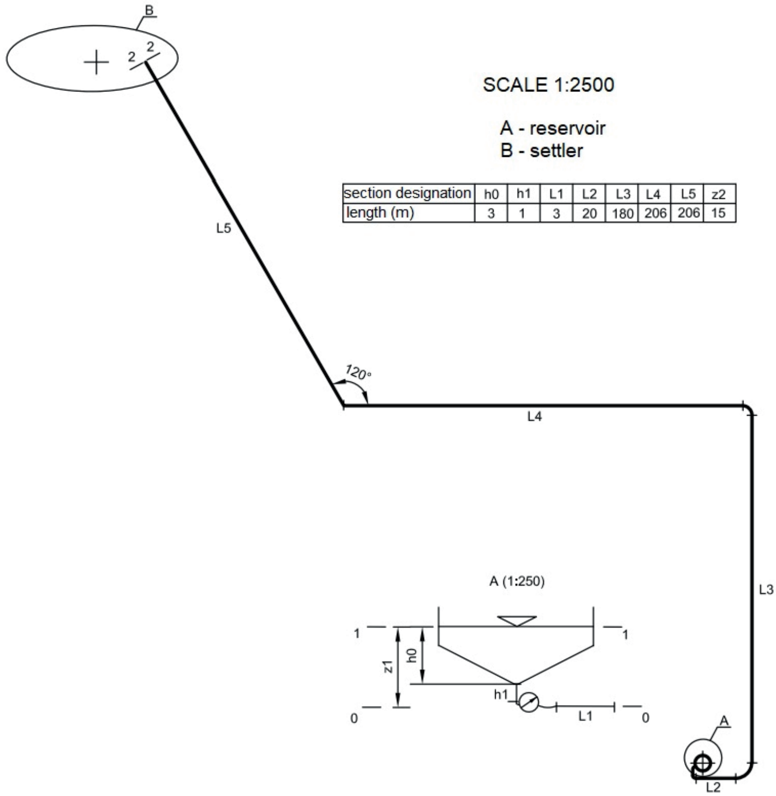

Figure 2 Scheme of the pipeline with chosen cross sections 1-1 and 2-2.

Figure 3 presents the predicted correlation between pressure losses and bulk velocity for several solid concentrations, with/without deflocculant.

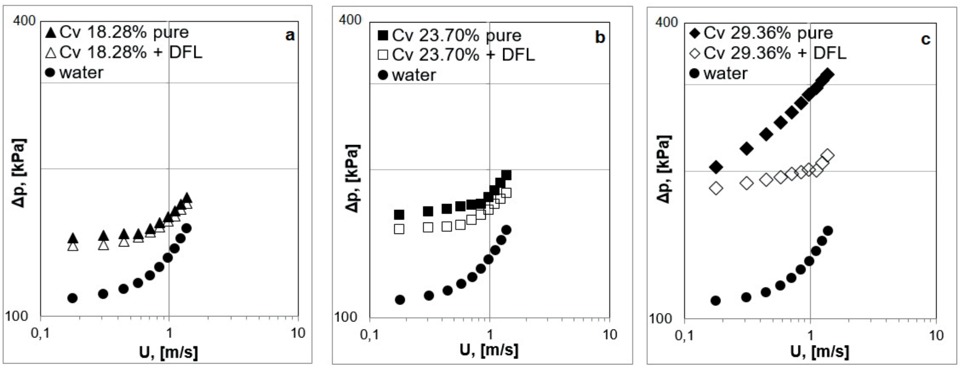

Figure 3 Correlation between pressure losses and the bulk velocity of lime slurries with volume concentrations of (a) 18.28%, (b) 23.70%, and (c) 29.36% in a pipeline.

The results of the pressure drop calculations versus bulk velocity for slurries with chosen concentrations, with and without the addition of deflocculant, are presented in Fig.3 in a double logarithmic arrangement of the axes. The curves presented in Fig. 3 show an increasing trend of the pressure drop with an increase in the solid concentration. However, this increase is smaller for slurries with the addition of deflocculant, which clearly indicates that the addition of a deflocculant reduces the friction loss, which is a result of the decreased viscosity of the slurry.

By analysing the data presented in Fig. 3 it is seen that there is a range of bulk velocities in which the data for Δp change linearly, which is especially apparent in Fig. 2c. In such a case, we can say that in the solid concentration slurry C_v=29.36%, which contains deflocculant, and flows in a pipe with d = 200 mm, the laminar flow appears for bulk velocities up to about 0.9 m/s. Figure 3 also indicates that for slurries with solid concentrations equal to 18.28% and 23.70% in the laminar flow range, the reduction in friction is greater, while in the turbulent flow range, the reduction in the friction factor is smaller. These observations confirm the general principle of drag reduction for the flow of non-Newtonian liquids if a proper deflocculant is applied.

4. DISCUSSION

The approach to laminar flow stability of the slurries with the yield stress was made by Ryan and Johnson, (1959), and Hanks, (1963). The Reynolds number for the Bingham plastic fluids is given by:

$$N_{\mathrm{Re}} = \frac{\rho_m \cdot U_s \cdot d}{\eta_p} \tag{5}$$

The critical value of the Reynolds number for Bingham plastic liquids that describes the transition from laminar to turbulent flow for the slurry tested can be calculated from the following equation, proposed by Hanks (1963):

$$(N_{\mathrm{Re}})_{\mathrm{cr}} = \frac{1 - \frac{4}{3}\phi_c + \frac{\phi_c^4}{3}}{8 \cdot \phi_c} \cdot N_{He} \tag{6}$$

where the parameter $\phi_c = \tau_y/(\tau_w)_c$ and the Hedström number are calculated as follows (Hedström, 1952):

$$N_{\mathrm{He}} = \frac{\rho \cdot d^2 \cdot \tau_y}{\eta_p^2} \tag{7}$$

and

$$\frac{\phi_{\mathrm{c}}}{(1-\phi_c)^3} = \frac{N_{He}}{16800} \tag{8}$$

For laminar flow of Bingham plastic fluids, the determination of the Fanning - friction factor (f_l) is given by (Chhabra and Richardson, 2008):

$$f_L = \frac{16}{N_{\text{Re}}} \cdot \left[1 + \frac{1}{6} \cdot \frac{N_{He}}{N_{\text{Re}}} - \frac{1}{3} \cdot \frac{N_{He}^4}{f_L^3 \cdot N_{\text{Re}}^7} \right] \tag{9}$$

Expression (9) is known as the Buckingham–Reiner equation and is recommended in the literature to describe the laminar flow of Bingham plastic fluids because of its simple form: the components of the equation are functions of the Reynolds and the Hedström numbers.

For turbulent flow, the friction factor (f_T) can be represented by the empirical expression of Darby and Melson (1981) and modified by Darby (1992):

$$f_T = 10^a \cdot N_{\text{Re}}^{-0.193} \tag{10}$$

where a is defined by the expression:

$$a = -1.47 \cdot \left[1 + 0.146 \exp(-2.9 \cdot 10^{-5} \cdot N_{He}) \right] \tag{11}$$

The friction factor for the flow of Bingham plastic liquids in smooth pipes can be calculated for any Reynolds number from the equation (Chhabra and Richardson, 2008):

$$f = \left(f_L^m + f_T^m \right)^{1/m} \tag{12}$$

where

$$m = 1.7 + \frac{40000}{N_{\text{Re}}} \tag{13}$$

The parameter f that determines the friction factor for Bingham plastic liquids can be calculated from the expression (12), according to the method presented by Chhabra and Richardson (2008) and Darby and Melson (1981).

The presented equations allow satisfactory values of pressure drop under turbulent conditions to be estimated for pipeline diameters less than 335mm, the Reynolds number $\leq 3.4 \cdot 10^5$ and the Hedström number in the range of $1000 \leq N_{He} \leq 6.6 \cdot 10^7$, which is consistent with the data presented in the book of Chhabra and Richardson, (2008). Compliance of the values of the lime slurry parameters with the applicability range of the above equations appears to confirm the usefulness of the proposed method.

The transition from laminar to turbulent flow in the case of non-Newtonian fluids does not occur suddenly, but gradually, in a certain transition zone, which in various studies is located in the Bingham Reynolds number range from 2000 to 10000. Available experimental results suggest much higher values for the critical Reynolds number for viscoplastic liquids (Kembłowski, 1973), (Myers et al., 2017), (Rudman et al., 2004), and (Wilson and Thomas, 2006).

The pipe velocity for laminar flow breakdown was calculated from Equation (5). The velocity values obtained from Equation (5) were adopted as the critical velocity of the laminar-turbulent transition for the tested lime slurries (Us_{cr}).

Table 2 presents the critical values of the Reynolds number obtained using Eqn (6) and the Hedström number, the parameter ϕ_c and the maximum permissible velocity for the flow of the tested Bingham plastic fluids, calculated from the given formulas.

Table 2

Parameters in the transition from laminar to turbulent flow regime of tested lime slurries

C_v (%)	$(N_{Re})_{cr}$	N_{He}	φ_c	Us_{cr} (m/s)
18.28 pure	$1{,}42 \cdot 10^4$	$8{,}09 \cdot 10^5$	0.750	0.368
18.28 + DFL	$1{,}26 \cdot 10^4$	$5{,}84 \cdot 10^5$	0.725	0.231
23.70 pure	$1{,}34 \cdot 10^4$	$6{,}92 \cdot 10^5$	0.739	0.524
23.70 + DFL	$1{,}25 \cdot 10^4$	$5{,}59 \cdot 10^5$	0.721	0.310
29.36 pure	$1{,}12 \cdot 10^4$	$4{,}13 \cdot 10^5$	0.695	1.393
29.36 + DFL	$9{,}7 \cdot 10^3$	$2{,}74 \cdot 10^5$	0.657	0.743

As can be seen from the parameters of the transition from laminar to turbulent flow of the lime slurries tested presented in Table 2, the values of the calculated parameters were lower for the slurries with the addition of deflocculant than in the pure slurries. This effect is the result of the decrease in the viscosity of the slurries after the addition of the deflocculant.

5. CONCLUSIONS

The article presents a method for determining the critical value of the Reynolds number and therefore the critical pipe velocity for laminar flow breakdown for a lime slurry with the addition of a deflocculant. In the work, the method for calculation of the friction factor for the tested Bingham plastic slurries was also presented.

The results of the research presented in the article may turn out to be particularly useful in cases where the tested liquid is a homogeneous or pseudo-homogeneous suspension or a multiphase system, for which it is crucial to determine the viscosity or rheological parameters. The description of the rheological properties of the slurries was determined on the basis of experimentally obtained flow curves using a rotational viscometer.

As a result of the addition of deflocculant to the fine dispersive lime slurry, the reduction of shear stresses and viscosity was obtained in the flow of the slurry in the pipeline. This effect facilitates the transport of slurries with a higher solid concentration, lower water consumption, and lower energy consumption.

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